Unit – I

Chapter – 1: Electric field intensity and potential

- **Electrostatics**:

The study of electric forces and electric fields that are produced by electric charges which are at rest is called electrostatics.

❖ Electric charge:

Charge is the property associated with matter due to which it produces and experiences electric and magnetic effects. Every atom is electrically neutral, containing equal no of electrons in orbits and protons in the nucleus. Charged particles can be created by disturbing the neutrality of an atom. Loss of electrons gives positive charge and gain of electrons gives negative charge to a particle.

Properties of electric charge:

1. Charge is a scalar quantity.
2. There are two types of charges i.e., positive charge and negative charge.
3. Like charges (positive – positive or negative – negative) repel each other while unlike charges (positive – negative) attract each other.
4. **Charge is conserved**: charge can neither be created and nor be destroyed. It can only be transferred from one point to another point.
5. **Charge is quantized**: The smallest charge that can exist in nature is the charge of the electron ($e = 1.6 \times 10^{-19} C$). The charge on any body is the integral multiple of e, i.e., $Q = \pm ne$ with $n = 1, 2, 3, \ldots$. The charge on any body can never be $\pm \frac{2}{3} e$, $\pm 17.2e, \ldots$
6. **Charge is additive in nature**: Total charge on a body is the algebraic sum of all the charges located anywhere on the body.

   Example: If a body has the charges 2C, - 5C, 4C, 6C, etc., then the total charge on the body is 2 – 5 + 4 + 6 = 7C. While adding the charges their sign must be taken into consideration.

❖ Coulomb's law:

Coulomb's law states that "***the force of attraction or repulsion between two stationary point charges is directly proportional to the product of the two charges and inversely proportional to the square of the distance between them***".

If q_1 and q_2 be two point charges separated from each other by a distance r in vacuum or air) then the force F acting between them is given by

$F \propto q_1 q_2$ and $F \propto \frac{1}{r^2}$

$F \propto \frac{q_1 q_2}{r^2}$

$$\boxed{F = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2}}$$

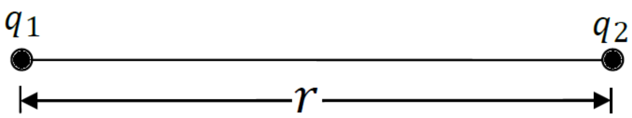


Fig 1.1

Where ε_0 is the permittivity of free space.

$\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 Nm^2/C^2$ and $\varepsilon_0 = 8.85 \times 10^{-12} C^2/Nm^2$

Coulomb's law in medium:

If instead of vacuum (or air) some medium (glass, paper, wax, oil etc.,) is placed between the charges, then the force F acting between them is given by

$$F = \frac{1}{4\pi\varepsilon_0 k}\frac{q_1 q_2}{r^2}$$

Where k = dielectric constant of the medium.

$\therefore F_{med} = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{r^2}$ $\quad\quad \because \varepsilon = \varepsilon_0 k$

Limitations of Coulomb's law:

Coulomb's law fails to explain the stability of the nucleus. The reason being that the nucleus consists of several protons all having positive charge and according to Coulomb's law they should repel each other. But we know that the nucleus has a stable identity. So the Coulomb's law fails.

- **Electric field**:

  The region surrounding an electric charge or a group of charges, in which another charge experiences a force is called electric field.

❖ **Intensity of electric field or electric field strength E :**

The intensity of electric field at a point in the field is defined as the force experienced by a unit positive charge placed at that point.

Let F be the force experienced by a test charge q_0 placed at a point in the field, then the intensity of electric field E at that point is given by

$E = \frac{F}{q_0}$ $\quad\quad$ Units: Newton/Coulomb or N/C

From the above equation, $F = Eq_0$

E is a vector quantity whose direction is in the direction of F.

If a unit positive charge $q_0(1C)$ is placed at a distance r from the point charge, then from Coulomb's law, the force on unit positive charge is given by

$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_0}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 \times 1}{r^2}$

$\frac{F}{1} = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r^2}$

$E = \frac{F}{q_0} = \frac{F}{1}$

$$\boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}}$$

❖ **Continuous charge distributions**: A system of closely spaced electric charges forms a continuous charge distribution.

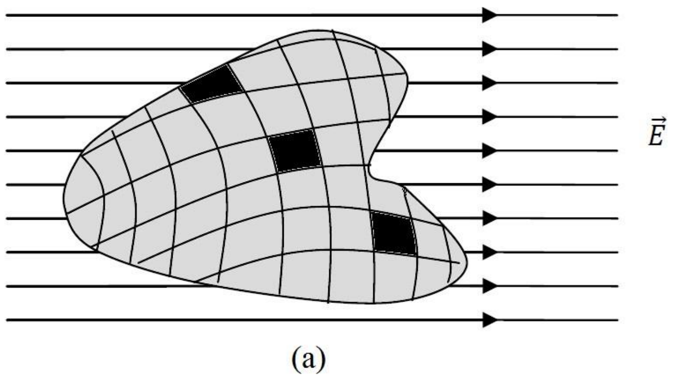
Linear charge distribution	Surface charge distribution	Volume charge distribution
In this distribution, charge is distributed on a line. Example: charge on a wire, charge on a ring.	In this distribution, charge is distributed on the surface. Example: charge on a conducting sphere, charge	In this distribution, charge is distributed in the whole volume of the body. Example: Solid uniformly charged object say a

The linear charge density $\lambda = \frac{\text{Charge}}{\text{Length}}$	on a sheet. The surface charge density $\varsigma = \frac{\text{Charge}}{\text{Area}}$	charged sphere. The volume charge density $\rho = \frac{\text{Charge}}{\text{Volume}}$

❖ **Electric flux**:

The no of lines of force passing through an area element which is making an angle with the direction of electric field is called electric flux Φ_E.

The scalar product, i.e., E · dS is defined as the electric flux for the surface.



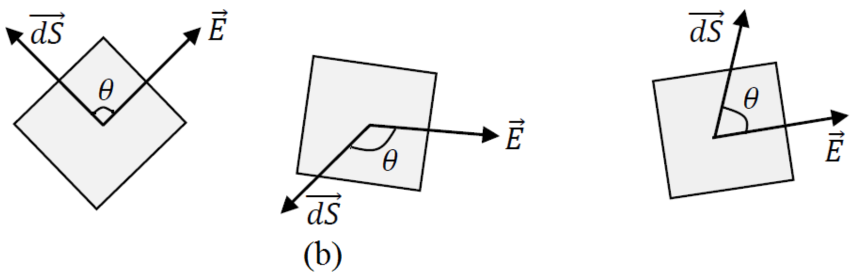


Fig 1.2 (a) Surface immersed in electric field
(b) Enlarged view of the three squares on surface

Consider a closed surface of area A in an electric field. Let the surface be divided into a no of elementary squares each of area dS. If E is the electric field acting on dS then the flux through dS is

$d\Phi_E = E \cdot dS$

The total flux through the entire surface is given by

$\Phi_E = E \cdot dS = E\ dS \cos\theta$ $\Phi_E = E$
$\cos\theta\ dS$

$\Phi_E = E \cos\theta\ A$

($\because dS = A$ = area of the surface)

$\Phi_E = EA\cos\theta$

The flux Φ_E of the electric field is measured by the number of electric lines of force that cut the surface.

Note: area elements

1. $\Phi_E = +ve$, If the lines of force point outward everywhere (Out flux)

2. $\Phi_E = 0$, If the lines of force are perpendicular to the area
3. $\Phi_E = -ve$, If the lines of force point inward everywhere (influx)

Example: Electric flux through cylinder.

Consider a cylinder of radius R immersed in a uniform electric field E parallel to its surface as shown in figure.

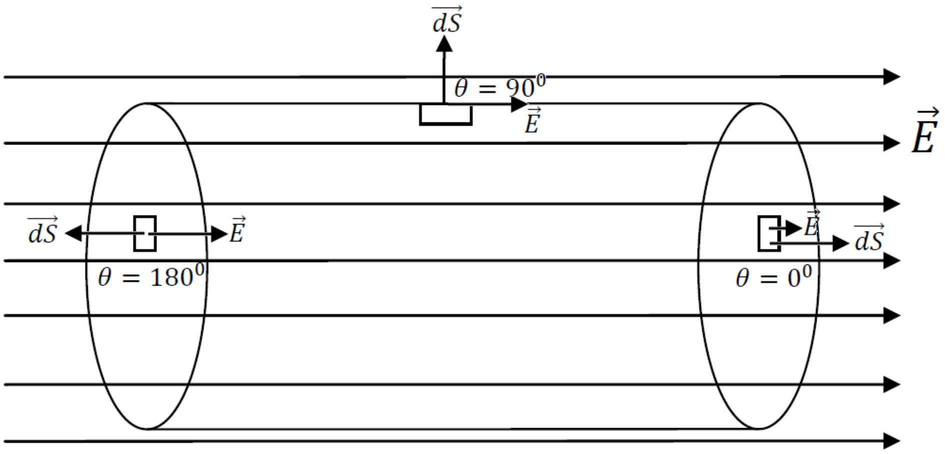


Fig 1.3

The flux through the entire cylinder is the sum of the fluxes through (a) left face (b) right face and (c) the cylindrical surface. Thus

$$\Phi_E = E \cdot dS = E \cdot dS_{(a)} + E \cdot dS_{(b)} + E \cdot dS_{(c)}$$

For the left face, the angle between E and dS is 180^0

$\therefore \quad E \cdot dS_{(a)} = EdS\cos 180^0 = -E \quad dS = -ES \quad$ where $S = \pi R^2$

For the right face, the angle between E and dS is 0^0

$\therefore \quad E \cdot dS_{(b)} = EdS\cos 0^0 = E \quad dS = +ES$

For the curved surface of the cylinder, the angle between E and dS is 90^0

$E \cdot dS_{(c)} = EdS\cos 90^0 = 0$

$\Phi_E = -ES + ES + 0$

Thus the flux through the entire cylinder is zero.

❖ **<u>Gauss's law</u>**:

Statement: Gauss's law states that total normal electric flux ϕ_E over a closed surface is $1\ \epsilon_0$ times the total charge Q enclosed within the surface. It is expressed mathematically as

$$\phi_E = \mathbf{E} \cdot \mathbf{dS} = E \cdot ds \cos\theta = \frac{1}{\epsilon_0} Q_{enc}$$

where ϵ_0 is the permittivity of free space.

Proof:

(i) when the charge is within the surface

Let a charge Q is placed at O within a closed surface of irregular shape. Consider a point P on the surface at a distance r from O . Now take a small area element dS around P. Let dS be normal vector of area element dS making an angle θ with the direction of electric field along OP.

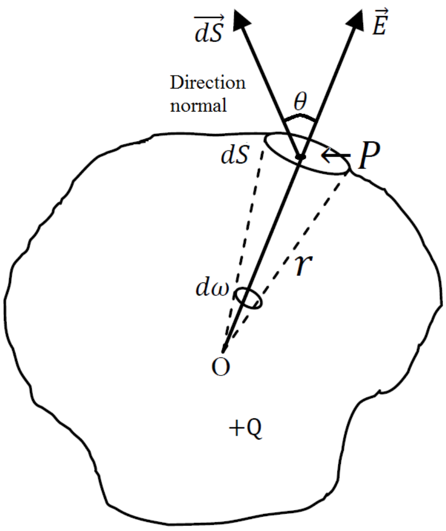


Fig 1.4

The electric flux $d\phi_E$ outwards through the area dS is given by

$d\phi_E = \boldsymbol{E} \cdot \boldsymbol{dS} = E dS \cos\theta$ ....(1)

θ is the angle between $\boldsymbol{E}$ and $\boldsymbol{dS}$

From Coulomb's law, the electric intensity E at a distance r from a point charge Q is given by $E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$..... (2)

From (1) and (2), $d\phi_E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2} dS\cos\theta = \frac{Q}{4\pi\epsilon_0}\frac{dS\cos\theta}{r^2}$

But $\frac{dS\cos\theta}{r^2}$ is the solid angle $d\omega$ subtended by dS at O.

$d\phi_E = \frac{Q}{4\pi\epsilon_0} d\omega$ .... (3)

The total flux ϕ_E over the entire closed surface is given by $\phi_E = \frac{Q}{4\pi\epsilon_0} \oint d\omega$

$\phi_E = \frac{Q}{4\pi\epsilon_0} \times 4\pi$ ($\because d\omega$ is the angle subtended by the whole surface at O. This is equal to 4π)

$$\boxed{\phi_E = \frac{Q_{enc}}{\epsilon_0}}$$ .... (4)

Note: When the closed surface encloses several charges like $+Q_1$, $+Q_2$, $+Q_3$,...... $-Q'_1$, $-Q'_2$, $-Q'_3$,....

The total flux is given by

$\phi_E = \frac{1}{\epsilon_0}\left(+Q_1 + Q_2 + Q_3 ...... - Q'_1 - Q'_2 - Q'_3 ...\right)$

$= \frac{1}{\epsilon_0} Q$, where Q is the algebraic sum of all charges.

(ii) when the charge is outside the surface

Let a point charge $+Q$ be placed at point O outside the closed surface as shown in fig. Now a cone of solid angle $d\omega$ from O cuts the surfaces dS_1, dS_2, dS_3, dS_4 at P, Q, R and S respectively.

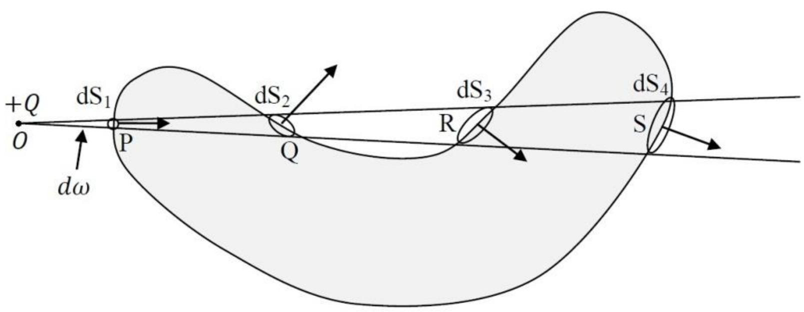


Fig 1.5

The electric flux for an outward normal is positive and for inward normal is negative. Therefore, the flux through areas dS_2 and dS_4 are positive while for dS_1 and dS_3 are negative.

The electric flux at P through area dS_1 $= -\frac{Q}{4\pi\epsilon_0} d\omega$

The electric flux at Q through area dS_2 $= +\frac{Q}{4\pi\epsilon_0} d\omega$

The electric flux at R through area dS_3 $= -\frac{Q}{4\pi\epsilon_0} d\omega$

The electric flux at S through area dS_4 $= +\frac{Q}{4\pi\epsilon_0} d\omega$

$\therefore$ Total electric flux $= -\frac{Q}{4\pi\epsilon_0} d\omega + \frac{Q}{4\pi\epsilon_0} d\omega - \frac{Q}{4\pi\epsilon_0} d\omega + \frac{Q}{4\pi\epsilon_0} d\omega = 0$

So the total electric flux over a closed surface due to an external charge is zero. This verifies Gauss's law.

❖ Differential form of Gauss's law:

According to Gauss's law, $\mathbf{E} \cdot d\mathbf{S} = Q \; \epsilon_0$ or $\epsilon_0 \; \mathbf{E} \cdot d\mathbf{S} = Q$ .... (1)

Let a charge be distributed uniformly over a volume V and ρ be the charge density. Then

$Q = \rho \, dV$ .... (2)

$\therefore \epsilon_0 \; \mathbf{E} \cdot d\mathbf{S} = \rho \, dV$ .... (3)

Apply Gauss divergence theorem to convert surface integral to volume integral

i.e., $_s \mathbf{E} \cdot d\mathbf{S} = {}_V div\, \mathbf{E}\, dV$ ....(4)

from (3) and (4), we get $\epsilon_0 \; {}_V div\, \mathbf{E}\, dV = \rho\, dV$ .... (5)

equating on both sides $\epsilon_0 div\, \mathbf{E} = \rho$

$div\, \mathbf{E} = \frac{\rho}{\epsilon_0}$ or $\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$

$$\boxed{\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}} \qquad \text{........... (6)}$$

Eq. (6) is the differential form of Gauss's law.

❖ Electric field due to a uniformly charged sphere:

Case (i): At a point outside the charged sphere ($r > R$):

Consider a sphere A of radius R with center O as shown in fig. Let q be a charge uniformly distributed over the sphere. Let P be a point outside the charged sphere which is at a distance r from the centre O of the sphere.

To find the electric field at P, construct a Gaussian surface of radius OP concentric with sphere A. The electric field at all points on the Gaussian surface is equal in magnitude and perpendicular to the surface.

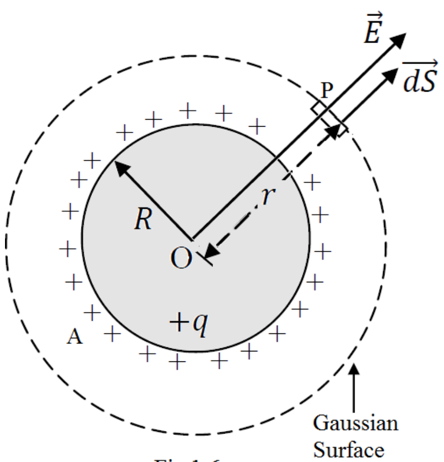


Fig 1.6

Let dS be a small area element of this surface and $d\phi_E$ is the flux passing through this element.

Therefore, $d\phi_E = E \cdot dS = E\, dS \cos 0^0 = E\, dS$ (∵ angle between E and dS is zero)

The electric flux through the entire Gaussian surface is given by

$$\phi_E = \oint E \cdot dS = E \oint dS = E\ 4\pi r^2 \qquad \text{......... (1)}$$

From Gauss's law, $\phi_E = E\, 4\pi r^2 = \frac{q}{\epsilon_0}$

$$\therefore\ E = \frac{1}{4\pi\epsilon_0} \times \frac{q}{r^2} \qquad \text{.......... (2)}$$

Case (ii): At a point on the surface $(r = R)$

When the point P lies on the surface of the charged sphere, then $r = R$. The electric field intensity is given by

$$E = \frac{1}{4\pi\epsilon_0} \times \frac{q}{R^2} \qquad \text{.......... (3)}$$

Case (iii): At a point inside the charged sphere $(r < R)$

Consider P be a point at a distance r from the centre inside the charged sphere. Let OP be the radius of Gaussian sphere and ρ is the charge density (charge per unit volume).

Now, to calculate the electric field E at P, the outward flux through a small portion of Gaussian surface is

$d\phi_E = E \cdot dS = E\ dS \cos 0^0 = E\ dS$ (∵ angle between E and dS is zero)

The electric flux through the entire Gaussian surface is given by

$$\phi_E = \oint E \cdot dS = E \oint dS = E\ 4\pi r^2$$

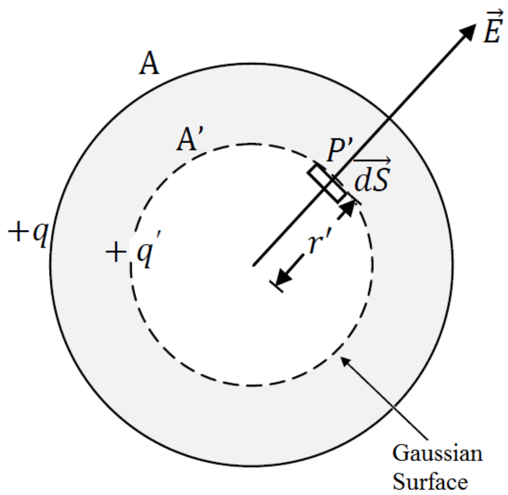


Fig 1.7

$$\rho = \frac{q}{\frac{4}{3}\pi R^3} = \frac{3q}{4\pi R^3}$$

The total charge enclosed by the Gaussian surface = volume enclosed by it × charge per unit volume

$$= \frac{4}{3}\pi r^3 \times \rho$$

$$\therefore \text{charge enclosed in Gaussian surface} = \frac{4}{3}\pi r^3 \times \frac{3q}{4\pi R^3} = q\left(\frac{r}{R}\right)^3 \qquad \text{.......... (4)}$$

From Gauss's law, $\phi_E = E\ 4\pi r^2 = \dfrac{\text{charge enclosed in Gaussian surface}}{\epsilon_0} = \dfrac{q}{\epsilon_0}\left(\dfrac{r}{R}\right)^3$

$$\therefore E = \frac{1}{4\pi\epsilon_0} \times \frac{qr}{R^3} \qquad \text{........... (5)}$$

This expression shows that the electric field E due to a uniformly charged sphere at an internal point is proportional to the distance r.

Graph:

- It is obvious that the electric field outside the sphere is inversely proportional to the square of the distance.
- It is maximum on the surface of the sphere.
- Electric field E due to a uniformly charged sphere at an internal point is proportional to the distance r.

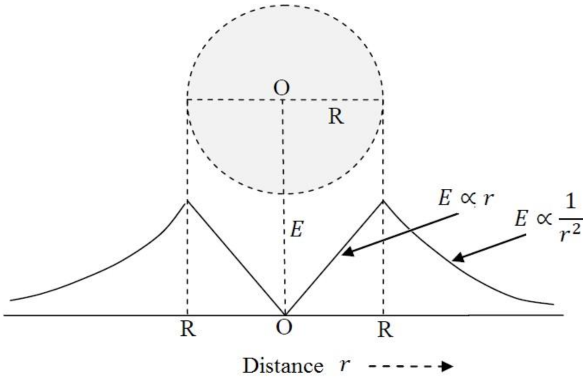


Fig 1.8

❖ **<u>Electric field due to hallow sphere or Electric field due to charged spherical shell</u>**:

Case (i): At a point outside the charged sphere ($r > R$):

Consider a sphere A of radius R with center O as shown in fig. Let q be a charge uniformly distributed over the sphere. Let P be a point outside the charged sphere which is at a distance r from the centre O of the sphere.

To find the electric field at P, construct a Gaussian surface of radius OP concentric with sphere A. The electric field at all points on the Gaussian surface is equal in magnitude and perpendicular to the surface.

Let dS be a small area element of this surface and $d\phi_E$ is the flux passing through this element.

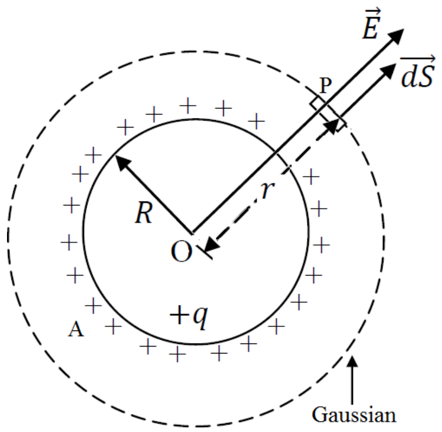


Fig 1.9

Therefore, $d\phi_E = E \cdot dS = E\,dS\cos 0^0 = E\,dS$ (∵ angle between E and dS is zero)

The electric flux through the entire Gaussian surface is given by

$\phi_E = E \cdot dS = E \; dS = E \; 4\pi r^2$ ……… (1)

From Gauss's law, $\phi_E = E\,4\pi r^2 = \frac{q}{\epsilon_0}$

$\therefore \; E = \frac{1}{4\pi\epsilon_0} \times \frac{q}{r^2}$ ……….. (2)

Case (ii): At a point on the surface ($r = R$)

When the point P lies on the surface of the charged sphere, then $r = R$. The electric field intensity is given by

$E = \frac{1}{4\pi\epsilon_0} \times \frac{q}{R^2}$ ……….. (3)

Case (iii): At a point inside the charged sphere ($r < R$)

In case charged spherical shell the charge resides only on the outer surface. Hence the charge enclosed by the Gaussian surface is zero.

From Gauss law,

$\phi_E = E \cdot dS = E \; dS = E \; 4\pi r^2$

$\phi_E = E\,4\pi r^2 = \frac{q}{\epsilon_0}$

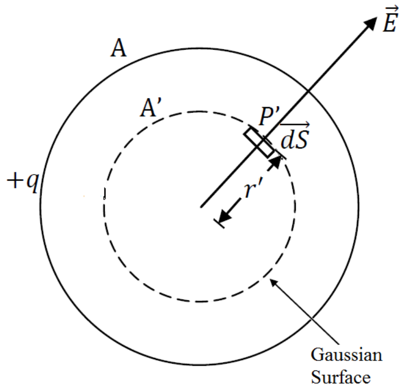


Fig 1.10

$E = 0 \qquad \because q = 0$

The electric field inside the charged conducting sphere or shell is zero.

Graph:

- It is obvious that the electric field outside the sphere is inversely proportional to the square of the distance.
- It is maximum on the surface of the sphere.
- Electric field E due to a charged sphere at an internal point is zero.

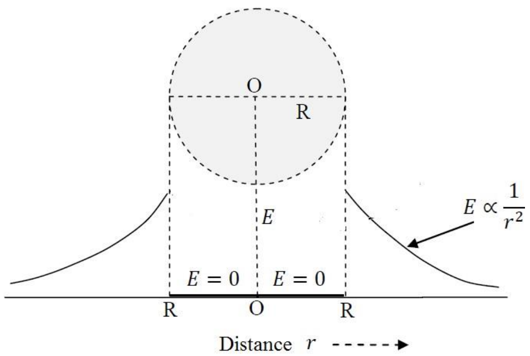


Fig 1.11

❖ **<u>Electric field due to infinite sheet of charge</u>:**

Consider a thin, non conducting, infinite sheet of charge. Let ς be the surface charge density (charge per unit area). Let P_1 be a point at a distance r from the sheet where the electric field is to be calculated. To calculate electric field, construct a Gaussian cylindrical surface by taking a point P_2 on other side of the sheet symmetrical about P_1. Let A be the area of cross section of cylindrical surface.

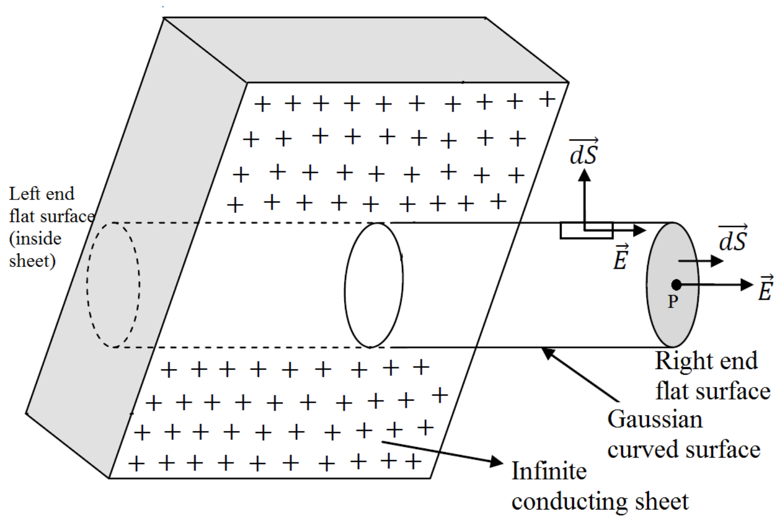


Fig 1.12

The electric flux through Gaussian surface is given by $\phi_E = \phi_{plane\ 1} + \phi_{curved\ surface} + \phi_{plane\ 2}$

$\phi_{plane\ 1} = \quad E \cdot dS = E \quad dS \cos 0^0 = EA$

$\phi_{curved\ surface} = E \cdot dS = E\ dS \cos 90^0 = 0$

$\phi_{plane\ 2} = \quad E \cdot dS = 0 \quad dS \cos 0^0 = 0$

$\phi_E = \phi_{plane\ 1} + \phi_{curved\ surface} + \phi_{plane\ 2} = EA + 0 + 0 = EA$

$\phi_E = EA$

According to Gauss's law, $\phi_E = \frac{q}{\epsilon_0}$

$EA = \frac{q}{\epsilon_0} = \frac{\varsigma A}{\epsilon_0}$ $\because \varsigma = \frac{q}{A}$

$\therefore E = \frac{\varsigma}{\epsilon_0}$

This expression shows that the magnitude of the electric filed is independent of the distance from the sheet.

❖ **Electric potential**:

Electric potential at a point in the electric field is defined as the work done by an external agent in carrying a unit positive charge from infinity to that point against the electric force of the field.

The SI unit of electric potential is *volt*.

Explanation:

The ratio of work done in taking a test charge from one point to other point in an electric field to the magnitude of the test charge is defined as the electric potential difference between these points.

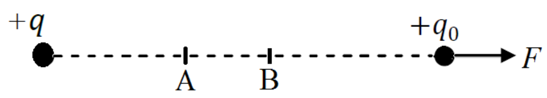


Fig 1.13

If W be the work done in moving the test charge q_0 from point B to point A, then the potential difference $V_A - V_B$ between A and B is expressed as

$$V_A - V_B = \frac{W}{q_0}$$

If point B is taken at infinity, then the electric potential V_B at infinite is zero.

$$V_A = \frac{W}{q_0}$$

❖ **Potential and Field strength**:

Let a uniform electric field $\boldsymbol{E}$ is set up by certain stationary charges (not shown) as shown in fig. Let q_0 be a charge moved by an external agent between two points A and B without acceleration in the electric field. The electric force on the charge due to electric field is $q_0\boldsymbol{E}$ which points downwards.

To move the charge upwards by external agent, an equal and opposite force $-q_0\boldsymbol{E}$ must be applied.

Let q_0 is moved through a small distance dl by the agent.

The work done dW by the agent is

$dW = F \cdot dl = -q_0 E \cdot dl$ ……… (1)

Therefore, the total work done

$W_{AB} = -q_0 \int_A^B E \cdot dl = -q_0 \int_A^B E \cdot dl$ ……… (2)

$V_B - V_A = \frac{W_{AB}}{q_0}$ ……… (3)

$\therefore$ Potential difference between two points A and B will be

$V_B - V_A = -\int_A^B E \cdot dl$ …….. (4)

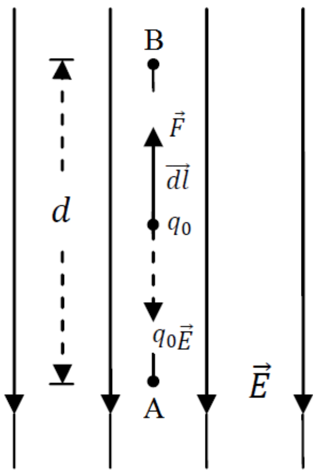


Fig 1.14

If the reference point A is taken at infinity, then $V_A = 0$.

Hence $$V_B = -\int_{\infty}^{B} E \cdot dl$$

So, the electric potential at a point in the electric field can be expressed as a line integral of the electric field.

- **<u>Potential due to a point charge</u>:**

Consider a point charge $+q$ as shown in fig. Its electric field is $\boldsymbol{E}$. Let A, B are two points at distances r_A and r_B from charge $+q$. To calculate the electric potential at point B, the charge is moved through a small distance $\boldsymbol{dr}$**.** The work done by external agent to move the charge through $\boldsymbol{dr}$ is given by

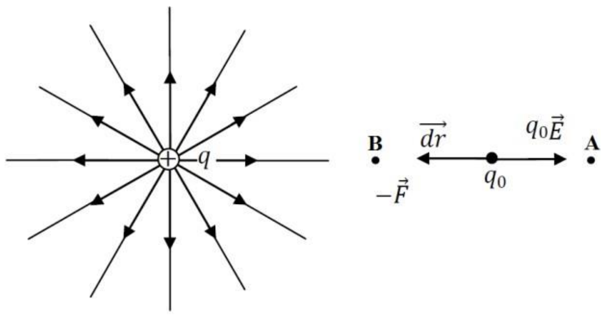


Fig 1.15

$$dW = q_0 E \cdot dr = q_0 E\, dr \cos 180^0 = -q_0 E\, dr$$

$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$$

$$\therefore dW = -\frac{1}{4\pi\epsilon_0}\frac{q q_0}{r^2} dr$$

Total work done in moving the charge from A to B is $W_{AB} = -\int_{r_A}^{r_B} \frac{1}{4\pi\epsilon_0}\frac{q\, q_0}{r^2} dr$

$$W_{AB} = -\frac{q q_0}{4\pi\epsilon_0}\left[-\frac{1}{r}\right]_{r_A}^{r_B} = \frac{q\, q_0}{4\pi\epsilon_0}\left[\frac{1}{r_B} - \frac{1}{r_A}\right]$$

So the potential difference between two points is $V_B - V_A = \frac{W_{AB}}{q_0} = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r_B} - \frac{1}{r_A}\right]$

Taking $r_A = \infty, V_A = 0$

$$V_B = \frac{1}{4\pi\epsilon_0}\frac{q}{r_B}$$

Drop the suffix B, $$V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$$

This expression shows that at a distance r on all sides of the charge q, the potential is the same.

- **<u>Potential due to charged spherical shell:</u>**

Case (i): when the point P lies outside the shell

Consider a conducting spherical shell with center O and radius R. The total charge on the sphere is q. Now we calculate potential V at point P at distance r from the center O. The relation between electric field and potential is given by

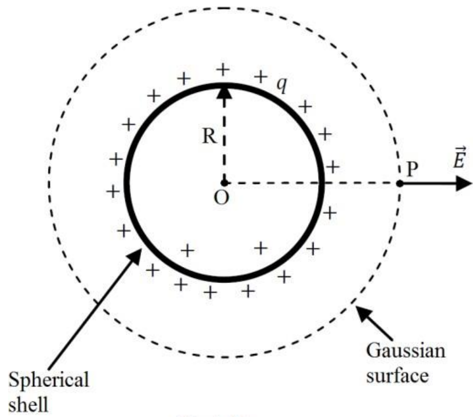


Fig 1.16

$$V_B - V_A = -\int_A^B E \cdot dr$$

Electric field due to charged spherical shell = $\frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$, $A=\infty, B=r$

$$V(r) = -\int_{\infty}^{r} \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}dr$$

$$V(r) = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r}\right]_{\infty}^{r}$$

$$V_{out} = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$$

Case (ii): at a point on the surface of sphere ($\boldsymbol{r = R}$)

In this case $r = R$, $V_{on} = \frac{1}{4\pi\epsilon_0}\frac{q}{R}$

Case (iii): at a point inside the sphere ($\boldsymbol{r < R}$)

We know that for spherical shell, $E_{out} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$

and $E_{in} = 0$

$$V_{in} = -\int_{\infty}^{r} E \cdot dr = -\int_{\infty}^{R} E_{out} \cdot dr - \int_{R}^{r} E_{in} \cdot dr$$

$$V_{in} = -\int_{\infty}^{R} E_{out} \cdot dr - 0 = -\int_{\infty}^{R} \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}dr$$

$$V_{in} = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r}\right]_{\infty}^{R}$$

$$V_{in} = \frac{1}{4\pi\epsilon_0}\frac{q}{R}$$

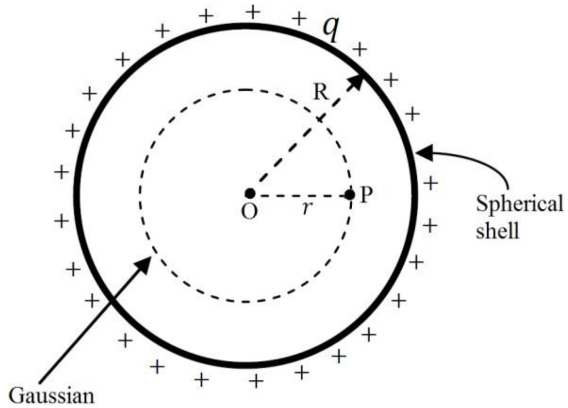


Fig 1.17

Graph:

- The potential inside the spherical shell is same as that on the surface.
- The electric potential outside the spherical shell is inversely proportional to the distance.

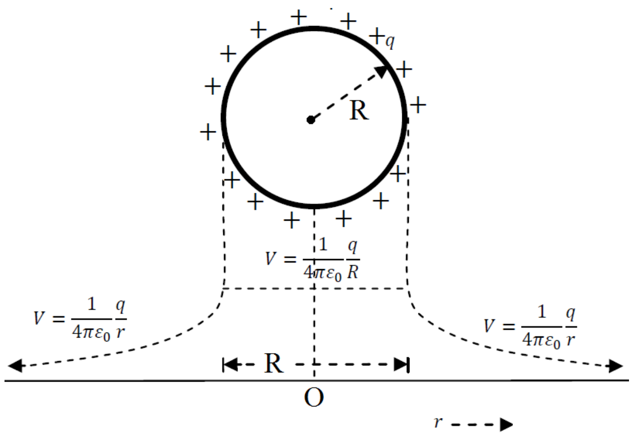


Fig 1.18

❖ Equipotential surfaces:

Equipotential surface in an electric field is a surface on which the potential is same at every point or the locus of all points which have the same potential is called equipotential surface. The potential difference between any two points on the equipotential surface is zero, hence no work is done in taking a charge from one point to another point. This is possible only when the charge is taken perpendicular to the field. So, the equipotential surface at every point is perpendicular to the field.

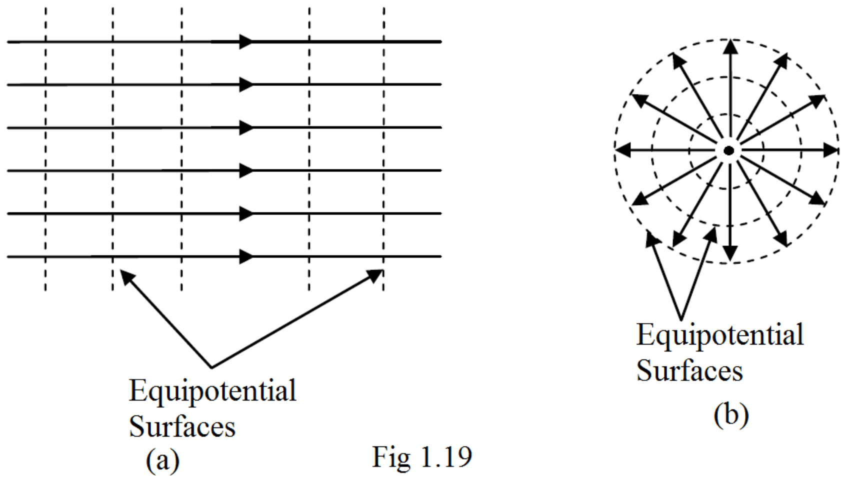


Fig 1.19

In case of uniform field, where the lines of force are straight and parallel, the equipotential surfaces are planes perpendicular to the lines of force as shown in fig. The equipotential surfaces are a family of concentric spheres for a point charge or sphere of charge.

Equipotential surfaces in electrostatics are similar to wavefronts in optics. The wavefronts in optics are the locus of all points which are in the same phase. These are the planes perpendicular to the direction of rays. On the other hand, the equipotential surfaces are perpendicular to the lines of force.

❖ **Important Questions**:

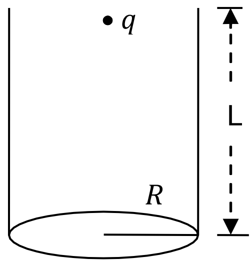
1. Define electric field intensity.
2. Explain about equipotential surfaces.
3. State and prove Gauss's law in electrostatics
4. Define electric field intensity. Calculate the electric field due to infinite conducting sheet of charge.
5. Calculate the electric field intensity due to uniformly charged sphere at points within and outside sphere.
6. Define electric potential. Derive expression for potential due to point charge.
7. Define electric potential. Calculate the electric potential due to spherical shell.

❖ **Problems**:

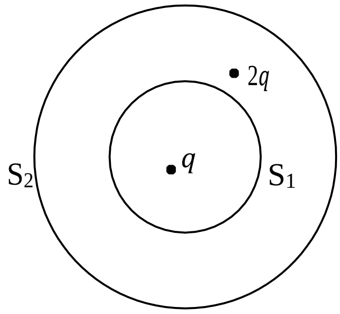
Based on Gauss's law:

1. If a point charge q is placed at the centre of a cube, what is the flux linked
   (a) with the cube
   (b) with each face of the cube?
2. A hemispherical body is placed in a uniform electric field $\boldsymbol{E}$. What is the flux linked with the curved surface, if field is (a) parallel to the base and (b) perpendicular to the base?
3. A charge q is placed at the centre of the open end of a cylindrical vessel of radius R

and length L. Calculate the flux through the surface of the vessel.



4. S_1 and S_2 are two parallel concentric spherical surfaces enclosing charges q and $2q$ respectively as shown in figure. What is the ratio of electric flux through S_1 and S_2?



Based on electric field intensity:

5. The intensity of electric field at a point 0.25m away a point charge is 1.44 N/C. Find the magnitude of charge.
6. A sphere charged to $80\mu C$ is placed in air. Find the electric field intensity at a point 20 cm from the centre of the sphere. Radius of the sphere R = 10 cm.

Based on electric potential:

7. A point charge is placed at A. The charge is 1.5×10^{-8} C. What are the radii of equipotential surfaces having a potential 15V and 30V .
8. The charge on a spherical conductor is 3×10^{-9} C. Radius of the conductor is 0.1m. find the potential. Take $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ N – m^2/C^2
9. What is the electric potential at the surface of nucleus gold? The radius of the nucleus is 6.6×10^{-15}m. The atomic number of gold is 79.
10. A spherical drop of water carrying a charge of 3×10^{-6} C has a potential 500V at its surface. What is the radius of the drop?
11. At distances of 5cm and 10cm from the surface of sphere, the potentials are 600V and 420V. Find the potential of its surface.

Unit – I
Chapter – 2: Dielectrics

❖ **Dielectrics**:

A dielectric or insulator is a material which does not contain free electrons or the number of such electrons is too low to constitute the electric current. In dielectrics, the electrons are tightly bound to the nucleus of atoms.

The dielectric does not conduct electricity on the application of electric field, the electrons may be able to move to and fro about their equilibrium positions but they do not have the vicinity of their atoms.

Examples: mica, glass, plastic etc.

USES:

1. In making high capacity condensers, paper and mica are used.
2. Quartz. Mica, glass, paffin etc are used for high insulation.
3. Ii helps in maintaining two large metal plates at very small separation.

❖ **Electric dipole**:

The arrangement of equal and opposite charges separated by a finite distance is called "electric dipole".

Electric dipole moment:

The product of magnitude of one charge q and distance between the two charges $2l$ is known as "Electric dipole moment".

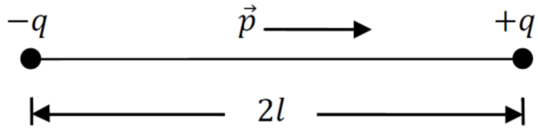


Fig 2.1

$\therefore$ Electric dipole moment $P = q \times 2l = 2ql$

This is a vector quantity, whose direction is along the axis of the dipole pointing from negative to positive charges.

❖ **Atomic view of dielectrics**:

- Atoms consist of positive and negative charges in equal magnitudes.
- The positive charge of the nucleus supposed to be concentrated at a single point called the centre of gravity of the positive charge.
- The negative charge of the electrons supposed to be concentrated at a single point called the centre of gravity of the negative charge.

- **Non – polar molecule**: when the two centers of gravity coincide, the molecule is known as non – polar molecule. Non – polar molecules have symmetrical structure and zero electric dipole moment.

  Example: H_2, N_2, CO_2, Benzene etc.
- **Polar molecule**: when the two centers of gravity do not coincide, the molecule is called is called

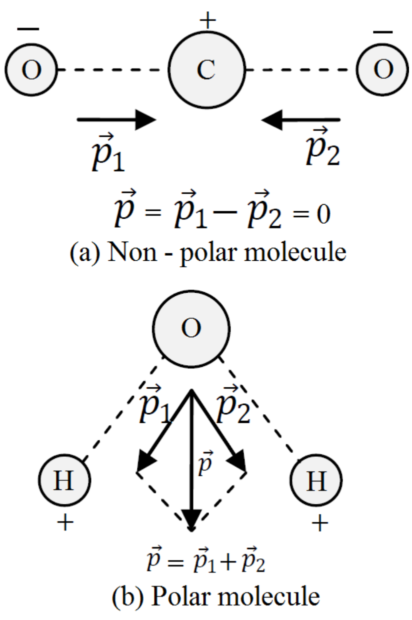


Fig 2.2

as polar molecule. Polar molecules have unsymmetrical structure and have a permanent dipole moment.

Example: H_2O, HCL, CO, NH_3 etc.

❖ **Non – polar dielectric (or) Polarization of dielectric**:

When a dielectric slab is place in an electric field, say between the plates of a charged capacitor, the centre of gravity of a positive charge is pulled towards the negative plate and centre of gravity of negative charged pulled towards the positive plate.

The process of separating positive and negative charges within the dielectric when placed in electric field is known as dielectric polarization.

The dielectrics which are polarized only when they are placed in an electric filed are called non-polar dielectrics.

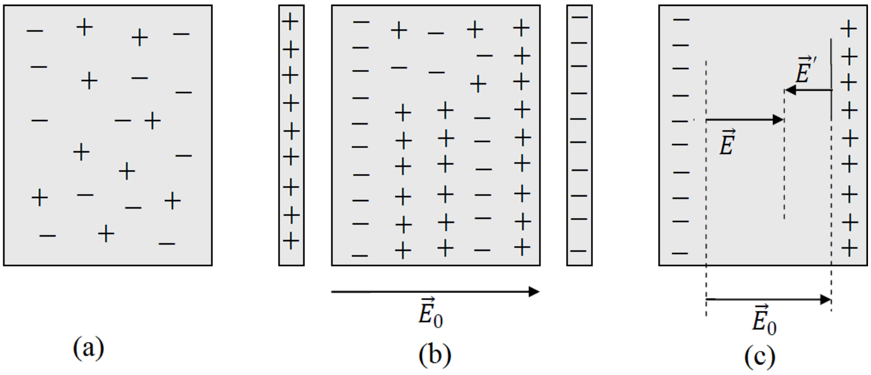


Fig 2.3

Fig (a) shows the random distribution of plus and minus charges in non-polar dielectric.

Fig (b) shows the surface charges appear (positive charge on the surface and negative charge on the other surface) when dielectric placed in electric field E_0.

Fig (c) shows the induced surface charges appear in such a way that the electric field set up by them E' opposes external filed E_0.

$$\therefore E = E_0 - E'$$

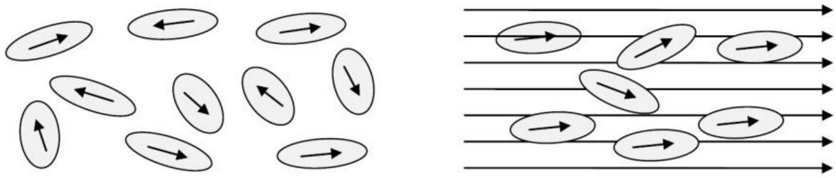
Thus, if the dielectric is placed in an electric field, induced surface charge appear which tend to weaken the original field within the dielectric.

❖ **Polar dielectric in electric field**:

Polar dielectrics have permanent dipole moments with their random orientations as shown in fig (a). In the presence of an electric field, the partial alignment of dipoles takes place as shown in fig (b). The alignment increases with the increase of electric field or with the decrease of temperature.

The dipole moment of a polar molecule in an electric field is $P_p + P_i$

P_p – permanent dipole moment, P_i - induced dipole moment



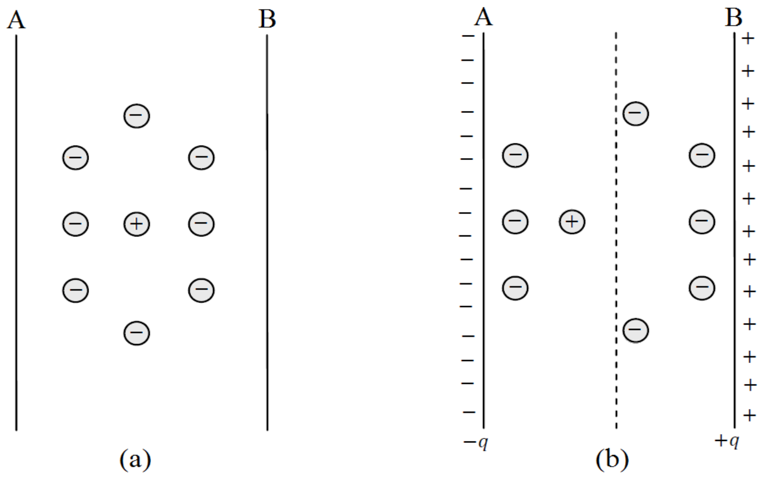
(a) (b)

Fig 2. 4

Thus the non-polar molecules in an electric field become induced dipoles while polar molecules are re-oriented with dipole moment increased.

❖ **<u>Dielectric Polarization & Charge density</u>**:

Let a dielectric slab (glass) is placed between the parallel plates of a capacitor. Fig (a) shows the electronic structure of an atom when the two plates are not charged. When the plates are charged, an electrostatic field is established between the two plates as shown in fig (b).



(a) (b)

Fig 2.5

Now the electrons are attracted towards the positive plate and positively charged nucleus is attracted towards negative plate of the capacitor. In this way, the dielectric is slab for be polarised and the distorted atom is called "Electric dipole" and every electric dipole has an "Electric dipole moment"

"***The electric dipole moment per unit volume is called as dielectric polarisation (P)***".

Suppose a dielectric slab of area of cross – section A and length l is placed in an electric field as shown in fig 2.6. Let the

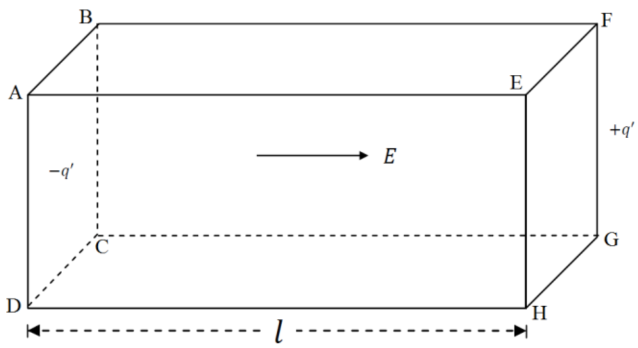


Fig 2.6

induced charges on faces ABCD and EFGH is $-q'$ and $+q'$ respectively.

The dipole moment $p = q' \times l = q'l$

Volume of the slab = Al

Dielectric polarization $P = \frac{dipole\ moment}{Volume}$

$P = \frac{q'l}{Al} = \frac{q'}{A}$ (charge per unit area i.e., charge density)

Hence dielectric polarization is numerically equal to the surface charge density.

❖ **<u>Dielectric constant & Susceptibility</u>**:

- **<u>Dielectric constant</u>:**

Based on Capacitance:

The ratio of the capacitance of a condenser with dielectric to the capacitance of the same condenser without dielectric is defined as dielectric constant.

$$k = \frac{C\ \text{(Capacitance of a condenser with dielectric)}}{C_0\ \text{(Capacitance of a condenser without dielectric)}}$$

Based on potential difference:

The potential difference V_d between the plates of the capacitor filled with dielectric is smaller than the potential difference V_0 without dielectric.

$$k = \frac{V_d}{V_0}$$

Based on force:

From Coulomb's law in free space $F_0 = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$

From Coulomb's law in medium $F_{med} = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{r^2}$

$$\frac{F_{med}}{F_0} = \frac{\varepsilon_0}{\varepsilon} = \frac{1}{k}$$

$k = \frac{\varepsilon}{\varepsilon_0}$ and $k = \frac{F_0}{F_{med}}$

From the above expression,

The dielectric constant is defined as the ratio of permittivity of the medium to the permittivity of the free space.

The dielectric constant is defined as the ratio of force between two charges in air or vacuum to the force between the same charges in dielectric medium.

The value of $k = 1$ for vacuum and $k = \infty$ for metals.

- **<u>Susceptibility</u>**:

When a dielectric is placed in electric filed is polarized. The polarization vector P is proportional the electric filed E.

$P \propto E$ or $P = \chi E$

Where the constant of proportionality χ is known as electric susceptibility.

The ratio of dielectric polarization to the electric intensity is defined as electric susceptibility.

$\chi = \frac{P}{E}$

❖ **<u>Gauss's law in dielectrics</u>**:

The Gauss's law states that the electric flux $\Phi_E$through any closed surface is 1 ε_0 times the charge enclosed by the surface.

$$\Phi_E = E \cdot dS = \frac{q}{\varepsilon_0}$$

Consider a parallel plate capacitor (a) without and (b) with dielectric as shown in fig. The charge q on the plates is same. The Gaussian surfaces are also shown in the figure.

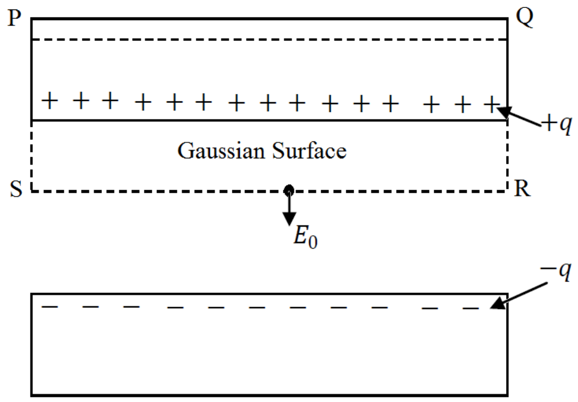


Fig 2.7 (a)

When no dielectric is present [fig 2.7(a)], then by Gauss's law

$$E \cdot dS = \frac{q}{\varepsilon_0}$$

$$E_0 \; dS = \frac{q}{\varepsilon_0}$$

$$E_0 A = \frac{q}{\varepsilon_0}$$

$$E_0 = \frac{q}{\varepsilon_0 A} \qquad \ldots\ldots\ldots (1)$$

When the dielectric is placed between the plates of the capacitor [fig 2.7(b)], the net charge within the Gaussian surface P'Q'R'S' is $q - q'$, where q' is the induced surface charge. Then by Gauss's law

$$E \cdot dS = \frac{q - q'}{\varepsilon_0}$$

$$EA = \frac{q - q'}{\varepsilon_0}$$

$$E = \frac{q}{\varepsilon_0 A} - \frac{q'}{\varepsilon_0 A} \qquad \ldots\ldots (2)$$

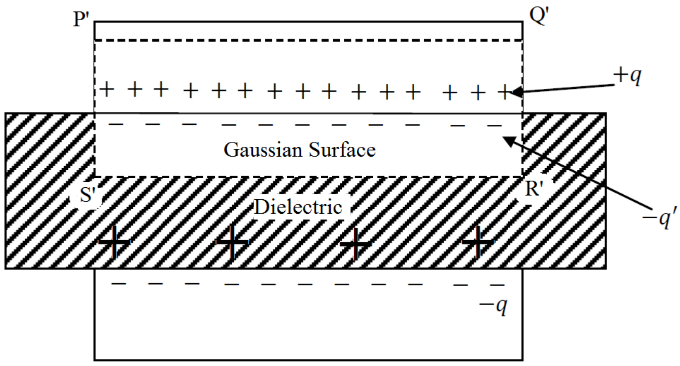


Fig 2.7 (b)

E is less than E_0 because induced charges produce their own field which opposes the original field

$$\frac{E_0}{k} = \frac{q}{\varepsilon_0 A} - \frac{q'}{\varepsilon_0 A} \qquad \because E = \frac{E_0}{k}$$

Substituting the value of E_0 from eq (1),

$$\frac{q}{\varepsilon_0 A k} = \frac{q}{\varepsilon_0 A} - \frac{q'}{\varepsilon_0 A}$$

$$q' = q - \frac{q}{k} = q\left(1 - \frac{1}{k}\right)$$

Now Gauss's law with dielectric present can be expressed as

$$E \cdot dS = \frac{q}{\varepsilon_0} - \frac{q}{\varepsilon_0}\left(1 - \frac{1}{k}\right)$$

$$E \cdot dS = \frac{q}{\varepsilon_0} - \frac{q}{\varepsilon_0} + \frac{q}{\varepsilon_0 k}$$

$$\boxed{k\, E \cdot dS = \frac{q}{\varepsilon_0}}$$

This is Gauss's law in a dielectric.

❖ **Three electric vectors & their relation**:

The three electric vectors are

1. Electric intensity

2. Dielectric polarization
3. Electric displacement

1. **Electric intensity E**:

The electric intensity E at any point in the electric field is numerically equal to the force experienced by a unit positive charge placed at that point. The direction of E being the same as that of the field.

2. **Dielectric polarization P**:

The electric dipole moment per unit volume is called as dielectric polarization P.

3. **Electric displacement D**:

The electric displacement at a point is defined as the product of electric field strength E at that point and the permittivity of the medium ε.

$$\therefore \boldsymbol{D} = \varepsilon \boldsymbol{E} = k\varepsilon_0 \boldsymbol{E}$$

The unit of $\boldsymbol{D}$ is C/m^2

D is equal to the surface charge density σ of free charges $D = \frac{q}{A} = \sigma$

When a dielectric slab is placed between the plates of a parallel plate capacitor, the medium is polarised. The relation between the induced charge q' and the charge q on the plate of the capacitor is given by

$$\frac{q}{\varepsilon_0 Ak} = \frac{q}{\varepsilon_0 A} - \frac{q'}{\varepsilon_0 A}$$

$$\frac{q}{\varepsilon_0 A} = \frac{q}{\varepsilon_0 Ak} + \frac{q'}{\varepsilon_0 A}$$

$$\frac{q}{A} = \varepsilon_0 \frac{q}{\varepsilon_0 Ak} + \frac{q'}{A}$$

$$\frac{q}{k\varepsilon_0 A} = E \text{ and } \frac{q'}{A} = P$$

$$\therefore \frac{q}{A} = \varepsilon_0 E + P$$

$$\boxed{D = \varepsilon_0 E + P}$$

Where D is the electric displacement.

Key points:

1. **D** is connected with free charge only. It is not altered by the introduction of the dielectric. The lines of **D** begin and end on free charges.
2. **P** is connected with polarization charge only. The lines of **P** begin and end on polarization charges.
3. **E** is connected with all charges that are actually present whether free or polarization. **E** is reduced inside dielectric with fewer lines.

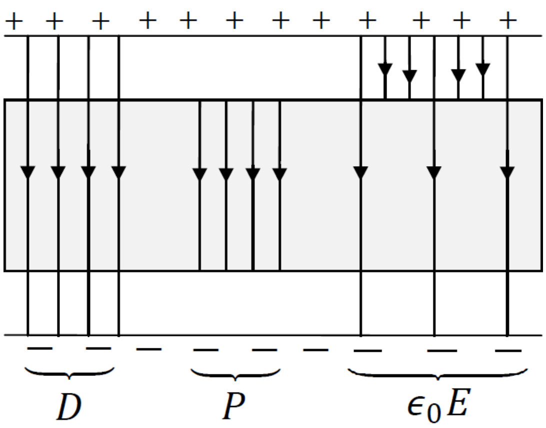


Fig 2.8

❖ **Important Questions**:

1. What are dielectrics? Explain polar and non – polar molecules.
2. What is meant by dielectric? Discuss the behaviour of a dielectric in an electric field from atomic point of view.
3. Define electric dipole moment.
4. Explain polarization and polarizability.
5. Define electric displacement vector, polarization and intensity of electric field.
6. Define and derive the relationship among D, E and P.
7. Define dielectric constant and susceptibility. Derive a relationship between dielectric constant and susceptibility.

❖ **Problems**:

1. The area of the plate of a parallel plate condenser is 100 cm^2. The distance between plates is 1 cm. A potential difference of 100 volt is applied. A slab of thickness 0.5 cm and dielectric constant 7 is placed between plates. Calculate the values of E, D and P in the air and dielectric.
2. The electric susceptibility of a medium is 948 × 10^{-11}. Calculate the permeability (or absolute permeability) and relative permeability.
3. If the dielectric constant of a medium is 3 and electric field intensity is 10^6 v/m, find the electric displacement D. ($\varepsilon_0 = 9 \times 10^{-12}$)
4. The thickness of dielectric between parallel plates of a condenser is 5 mm. Dielectric constant is 3. Electric field in the dielectric is 10^6 v/m. Calculate the surface charge density on the condenser plate, surface charge density on the dielectric, polarization, electric displacement and energy density.
5. The dielectric constant of water is 78. Calculate its electrical permittivity.
6. Calculate the value of dielectric constant, given permittivity in vacuum is 4 and permittivity of medium is 8.
7. The dielectric constant of medium is 4. Electric field in the dielectric is 10^6 v/m. Calculate electric displacement and polarization. ($\varepsilon_0 = 9 \times 10^{-12}$ F/m).
8. The electric susceptibility of a material is 36 × 10^{-12} C^2/N-m^2. Calculate the value of dielectric constant and absolute permittivity of the material. ($\varepsilon_0 = 9 \times 10^{-12}$ F/m).
9. The permittivity of diamond is 1.46 × 10^{-10} C^2/ N – m^2. Compute the dielectric constant and the electric susceptibility of diamond. $\varepsilon_0 = 8.9 \times 10^{-12}$ C^2/ N – m^2.
10. The electric susceptibility of a material is 35.4 × 10^{-12} C^2 / N – m^2. What are the values of dielectric constant and the permittivity of the material?
11. The dielectric constant of helium at 0^0 C is 1.000074. Find its electrical susceptibility at this temperature.
12. The dielectric constant of a medium is 3.5. Find its permittivity and susceptibility.
13. A NaCl crystal is subjected to an electric field of 1000 V/m. The resulting polarization is 4.3 × 10^{-8} C/m^2. Calculate the electronic polarizability.
14. Calculate the electronic polarizability of an Argon atom, given k = 1.0024 at N.T.P and N = 2.7 × 10^{35} atoms/m^3.
15. A solid elemental dielectric, with density 3 × 10^{28} atoms/m^3 shows an electronic polarizability of 10^{-40} farad m^2. Calculate the dielectric constant of the material.

Unit – II

Chapter 3: Electric and Magnetic Fields

❖ Force on a charged particle moving in a magnetic field:

If a charged particle of charge q moving with a velocity v enters into a magnetic field of induction B then it experience force given by

$$F = q\ v \times B$$

$$F = qvB\sin\theta$$

Where θ is the angle between v and B.

- The direction of F is given by Flemings left hand rule.
- If the charged particle is at rest $v = 0$ in magnetic field, then no force acts on it.

❖ Biot – Savart's law:

In 1820, Biot and Savart performed a series of experiments to study the magnetic field produced by various current carrying conductors. They obtained a relation by means of which **B** can be calculated at any point in space around a current carrying conductor. This relation is called as Biot and Savart law.

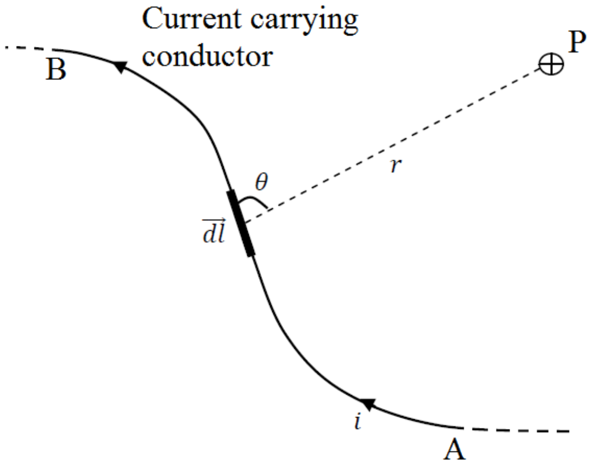


Fig 3.1

Let i be the current flowing through a conductor AB and P be a point at a distance r from the current element. According to Biot and Savart, the field B at any point can be calculated by dividing the conductor into no of infinitesimal current elements. Let dl be the length of one such element and dB be the magnetic field due to dl at a point P distance of r from it.

Biot and Savart observed that

(i) It is directly proportional to the current i flowing through the conductor $\boldsymbol{dB \propto i}$

(ii) It is directly proportional to the length of the element taken $\boldsymbol{dB \propto dl}$

(iii) It is directly proportional to the sine of the angle θ between length element and the line joining the element to the point P $\boldsymbol{dB \propto \sin\theta}$

(iv) It is inversely proportional to the square of the distance r of the point P from the element dl $\boldsymbol{dB \propto \frac{1}{r^2}}$

Combining all these factors $dB \propto \frac{i\,dl\sin\theta}{r^2}$

$$dB = \frac{\mu_0}{4\pi}\frac{i\,dl\sin\theta}{r^2}$$

Where $\frac{\mu_0}{4\pi}$ is proportionality constant, μ_0 - Permeability of free space.

The unit of dB is $weber/m^2$ (or) Tesla.

The resultant field at P is $B = \int dB$

- **Applications of Biot – Savart law**:
- **Magnetic field due to a long straight conductor (wire) carrying current**:

Consider an infinitely long wire placed in vacuum and carrying a current i. To calculate magnetic field B at a point P distance R from the centre O of the wire. We divide the wire into no of infinitesimal current elements. Consider one such element AB of length dl.

Let r be the distance of the element from the point P . θ the angle between dl and r.

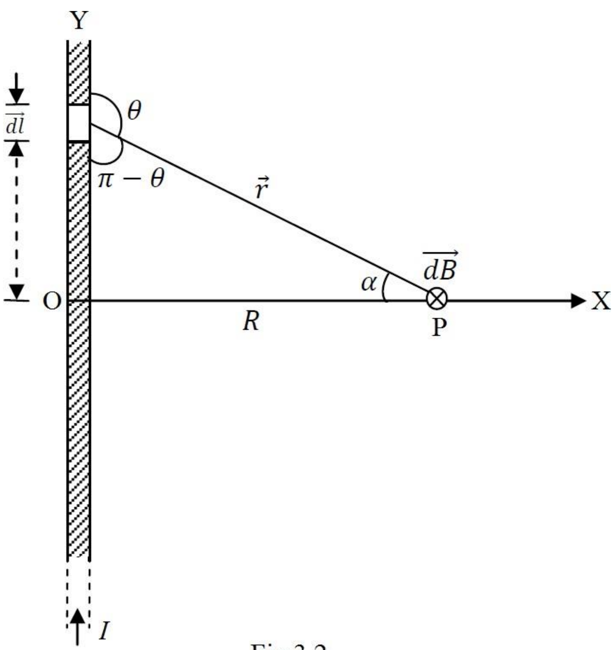


Fig 3.2

Magnetic field due to current element AB at P is $dB = \frac{\mu_0}{4\pi}\frac{i\, dl \sin\theta}{r^2}$

Magnetic field due to whole conductor

$B = \int_{-\infty}^{\infty} dB$

$B = \frac{\mu_0 i}{4\pi}\int_{-\infty}^{\infty}\frac{\sin\theta\, dl}{r^2}$

From figure $\sin\theta = \sin(\pi - \theta) = \frac{R}{r} = \frac{R}{\sqrt{l^2+R^2}}$

$$B = \frac{\mu_0 i}{4\pi}\int_{-\infty}^{\infty}\frac{R\, dl}{(l^2+R^2)^{3/2}}$$

To evaluate the above integral put $l = R\tan\alpha$

$$dl = R\, sec^2\alpha\, d\alpha$$

Limits: $l = \infty \Rightarrow \tan\alpha = \infty \Rightarrow \alpha = +\frac{\pi}{2}$

$l = -\infty \Rightarrow \tan\alpha = -\infty \Rightarrow \alpha = -\frac{\pi}{2}$

$$B = \frac{\mu_0 iR}{4\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{R\sec^2\alpha}{\left(R^2\tan^2\alpha + R^2\right)^{3/2}}d\alpha$$

$$B = \frac{\mu_0 iR}{4\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{R\sec^2\alpha}{R^3\sec^3\alpha}d\alpha$$

$$B = \frac{\mu_0 i}{4\pi R}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos\alpha\, d\alpha$$

$$B = \frac{\mu_0 i}{4\pi R}\left|\sin\alpha\right|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = \frac{\mu_0 i}{4\pi R}\left[\sin\frac{\pi}{2} - \left(-\sin\frac{\pi}{2}\right)\right]$$

$B = \frac{\mu_0 i}{2\pi R}$ weber/m^2 or Tesla

❖ **Magnetic field on the axis of a circular loop**:

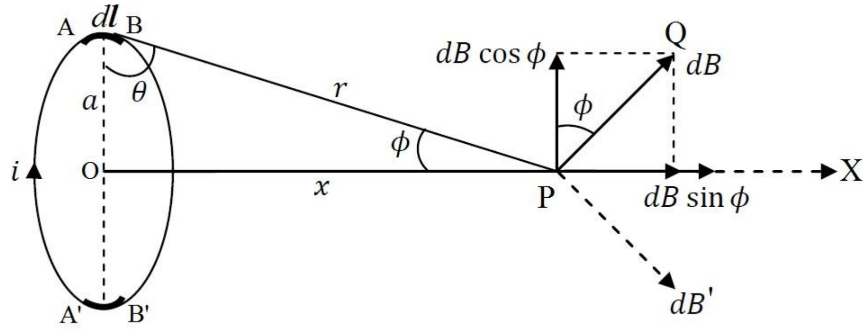


Fig 3.3

Consider a circular loop of radius $'a'$ and carrying a current i. Let P be a point on the axis of the coil at a distance x from the centre O. To calculate the field at P, consider a small element AB of length dl.

Let r be the distance of the element from the point P and θ = 90^0 be angle which the direction of current makes with the line joining the element to the point O.

$$dB = \frac{\mu_0}{4\pi}\frac{i\, dl \sin\theta}{r^2}$$

$$dB = \frac{\mu_0}{4\pi}\frac{i\, d}{r^2}$$

The vector dB at the point P, due to the element dl would be perpendicular to r.

This can be resolved into two components $dB \cos\phi$, $dB \sin\phi$

$dB \cos\phi$ → Perpendicular to the axis

$dB \sin\phi$ → along the axis

If we take another element $A'B'$ diametrically opposite to AB of same length.

$dB \cos\phi$ components will cancel out each other.

$dB \sin\phi$ components will add up along the axis.

Magnetic field along the axis = $dB \sin\phi$

$B = \frac{\mu_0 i\, dl}{4\pi\, r^2} \sin\phi$

$= \frac{\mu_0 i}{4\pi r^2}\, dl \sin\phi$

$= \frac{\mu_0 i}{4\pi r^2} dl \frac{a}{r}$ $\qquad \because \sin\phi = \frac{a}{r}$

$= \frac{\mu_0 i a}{4\pi r^3}\, dl$

$dl = 2\pi a$ (Circumference of the coil)

$B = \frac{\mu_0 i a}{4\pi r^3} \times 2\pi a$

$= \frac{\mu_0 i a^2}{2\,(a^2+x^2)^{3/2}}$ $\qquad \because r^2 = a^2 + x^2$

For N turns, $B = \frac{N\mu_0 i a^2}{2\,(a^2+x^2)^{3/2}}$ Weber/m^2 or Tesla

Different cases:

(i) At the centre of the coil $x = 0$. Thus, at the centre of the coil $B = \frac{N\mu_0 i a^2}{2a^3} = \frac{\mu_0 N i}{2a}$

(ii) At very far off from the loop $x \gg a$ and $(a^2 + x^2)^{3/2} \approx x^3$

$$B = \frac{\mu_0 N i a^2}{2x^3}$$

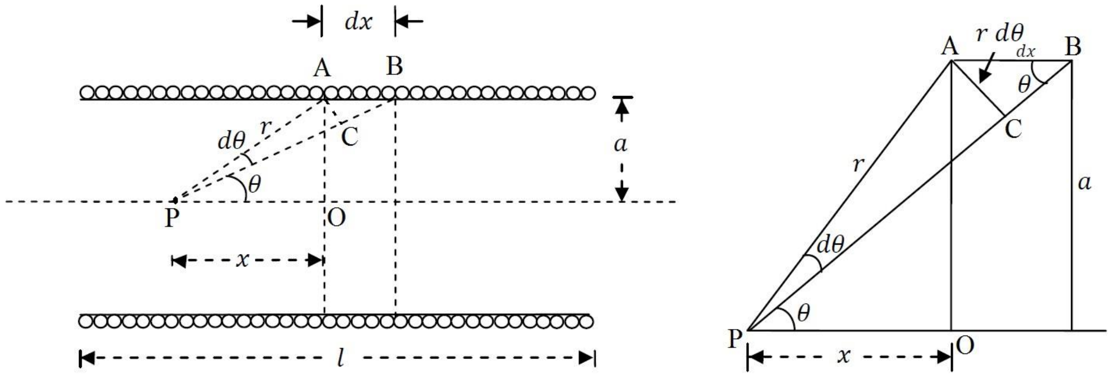
❖ **Magnetic field induction due to Solenoid**:

Consider a long solenoid of length l meter and radius a meter as shown in fig. Let N be the total no of turns in the solenoid. The no of turns n per meter will ne N/l. Let i be the current carried by solenoid.

Now we will calculate the field in the following cases:

(1) Field at an inside point
(2) Field at an axial point
(3) Field at the centre of the solenoid of finite length.

1. Field at an inside point



(a) (b) Enlarged View

Fig 3.4

Let us consider a point P inside the solenoid on the axis. To calculate B at point P, we divide the solenoid into a number of narrow equidistant coils. Consider one coil of width dx. The no of turns in dx is ndx. Let x be the distance of point P from the centre O of the coil.

The field at P due to elementary coil of width dx carrying current i is given by

$$dB = \frac{\mu_0\, n dx\, i a^2}{2(a^2+x^2)^{3/2}}$$

From ΔABC, $\sin\theta = \frac{r\,d\theta}{dx}$ or $dx = \frac{r\,d\theta}{\sin\theta}$

From ΔAPO, $r^2 = a^2 + x^2$

$$dB = \frac{\mu_0\, n\frac{r\,d\theta}{\sin\theta}\, i a^2}{2r^3}$$

$$= \frac{\mu_0 n i a^2 d\theta}{2r^2 \sin\theta}$$

$$dB = \frac{\mu_0 n i d\theta}{2\sin\theta}\left(\frac{a}{r}\right)^2 \qquad \because \sin^2\phi = \left(\frac{a}{r}\right)^2$$

$$= \frac{\mu_0 n i d\theta \sin^2\theta}{2\sin\theta}$$

$$dB = \frac{\mu_0 ni \sin\theta d\theta}{2}$$

The field induction at P due to whole solenoid can be obtained by integration between limits θ_1 and θ_2

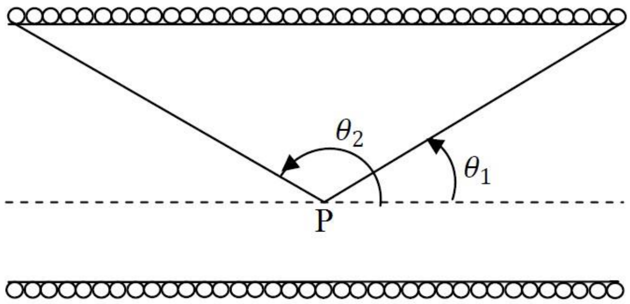


Fig 3.5

$$B = \int_{\theta_1}^{\theta_2} dB = \int_{\theta_1}^{\theta_2} \frac{\mu_0 ni \sin\theta d\theta}{2}$$

$$B = \frac{\mu_0 ni}{2}\left[-\cos\theta\right]_{\theta_1}^{\theta_2}$$

$$B = \frac{\mu_0 ni}{2}\left[\cos\theta_1 - \cos\theta_2\right]$$

At any axial point P, $\theta_1 = 0$, $\theta_2 = \pi$

$$B = \frac{\mu_0 ni}{2}\left[\cos 0 - \cos\pi\right] = \frac{\mu_0 ni}{2} 2$$

$$\boxed{B_{axial} = \mu_0 ni}$$

2. **Field at an axial end point**:
   In this case $\theta_1 = 0, \theta_2 = 90^0$

$$B = \frac{\mu_0 ni}{2}\left[\cos 0 - \cos 90^0\right] = \frac{\mu_0 ni}{2}$$

$$\boxed{B_{axial} = \frac{\mu_0 ni}{2}}$$

   This shows that field at either end is one half of its magnitude at the centre.

3. **Field at the centre of a solenoid of finite length**:
   Consider a point P is at the centre, i.e., it is at a distance of $l/2$ from either end.

$$\cos\theta_1 = \frac{l/2}{\left[a^2 + \left(\frac{l}{2}\right)^2\right]^{1/2}} = \frac{l}{\sqrt{4a^2+l^2}}$$

$$\cos(\pi - \theta_2) = \frac{l/2}{\left[a^2 + \left(\frac{l}{2}\right)^2\right]^{1/2}} = \frac{l}{\sqrt{4a^2+l^2}}$$

$$\cos(\pi - \theta_2) = \frac{-l}{\sqrt{4a^2+l^2}}$$

We know that $B = \frac{\mu_0 ni}{2}\left[\cos\theta_1 - \cos\theta_2\right]$

$$B = \frac{\mu_0 ni}{2}\left[\frac{l}{\sqrt{4a^2+l^2}} + \frac{l}{\sqrt{4a^2+l^2}}\right]$$

$$B = \frac{\mu_0 nil}{\sqrt{4a^2+l^2}} = \frac{\mu_0 Ni}{\sqrt{4a^2+l^2}}$$

$$\boxed{B = \frac{\mu_0 Ni}{\sqrt{4a^2+l^2}}}$$

This expression gives the field at the centre of the solenoid of finite length.

❖ **Hall effect**:

When a magnetic field is applied (along y – axis) perpendicular to a current (along x – axis) carrying conductor, then a potential difference is developed (along z – axis) between the points on opposite side of the conductor. This effect is known as Hall effect.

Explanation:

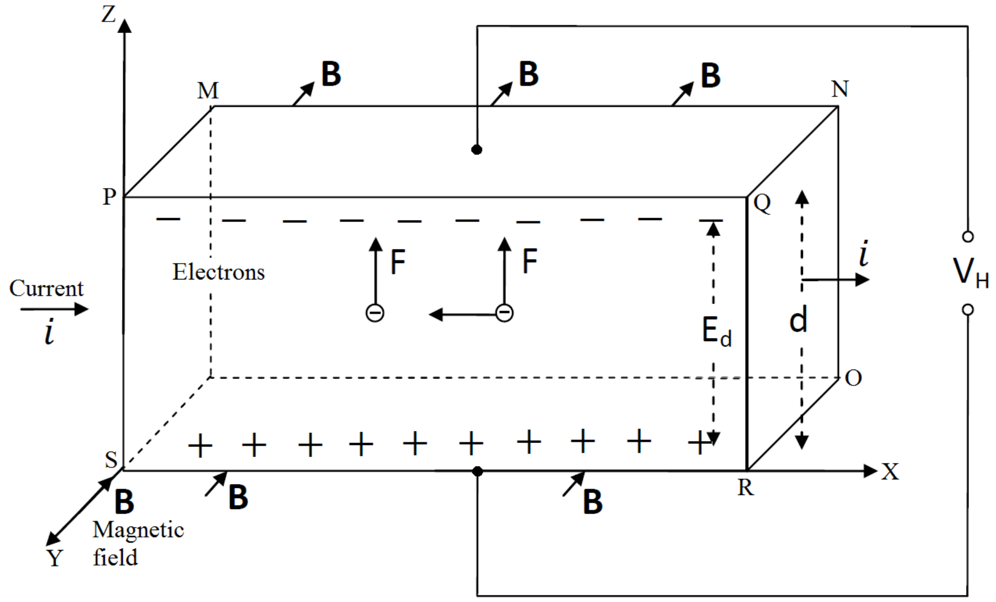


Fig 3.6

Consider a uniform, thick metal strip placed with its length parallel to x – axis. Let i be the current passed through the conductor along x – axis and a magnetic field **B** is established along y – axis. Due to the magnetic field, the charge carriers experience a force along z – axis. The direction of this force is given by Fleming's left hand rule. Hence, electrons will be accumulated on the upper surface of the strip, i.e., on face PQNM as shown in fig. Due to this fact the upper side will be negatively charged while the lower side will be positively charged. Thus, a transverse potential difference is created. This e.m.f is known as Hall e.m.f.

Hall field and Hall voltage:

When the equilibrium is reached, the magnetic deflecting forces on the charge carriers are equal to the electric forces due to electric field.

Magnetic force = $q\,\boldsymbol{v}_d \times \boldsymbol{B}$

Hall electric force = $q\boldsymbol{E}_H$ ($\boldsymbol{E}_H$ = Hall field)

Net force on the charge carriers becomes zero $q\,\boldsymbol{v}_d \times \boldsymbol{B} + q\boldsymbol{E}_H = 0$

$\boldsymbol{E}_H = -\,\boldsymbol{v}_d \times \boldsymbol{B} = -v_d\,B$

$E_H = -v_d\,B$

Drift velocity is related to the current density j by $v_d = \frac{j}{nq}$

n − no of charge carriers per unit volume

Hall field $E_H = \frac{1}{nq} jB$

If V_H is the Hall voltage in equilibrium, then $E_H = \frac{V_H}{d}$

Thus, measuring the potential difference V_H between the two faces, E_H can be calculated using the above equation.

Hall coefficient:

The ratio of Hall electric field E_H to the product of current density j and magnetic induction B is known as Hall coefficient. This is denoted by R_H.

$$R_H = \frac{E_H}{jB}$$

$$\frac{E_H}{jB} = \frac{1}{nq}$$

$$\boxed{R_H = \frac{1}{nq}}$$

The Hall coefficient is negative when the charge carriers are electrons and positive when the charge carriers are holes.

❖ **Applications of Hall effect**:

1. Hall effect gives the information about the sign of charge carriers in electric conductor. It is found that most metals have negatively charged electrons.
2. Hall effect is quite helpful in understanding the electrical conduction in metals and semiconductors.
3. Hall effect can be used to measure the drift velocity of the charge carriers.
   $$v_d = \frac{j}{nq}$$
4. The mobility of the charge carriers can be measured by the conductivity of the material and Hall electric field.

❖ **Important Questions**:

1. State and explain Biot and Savart law.
2. Calculate the intensity of magnetic field due to a long straight conductor carrying current.
3. Calculate the intensity of magnetic field at a point on the axis of a circular coil carrying current.
4. What is Hall effect? Mention its applications.
5. Explain Biot and Savart law. Calculate the intensity of magnetic field at a point on the axis of a circular coil carrying current.
6. State and explain Biot and Savart law. Calculate B due to long straight wire using it.
7. State and explain Boit – Savart law. Calculate B inside a long solenoid carrying current i. Show that the field at the ends of such a solenoid is half of that in the middle.
8. Define Hall effect. Derive the expression for Hall coefficient. Write the applications of Hall effect.

❖ **Problems**:

1. A long straight wire carries a current of 3.5 A. Find the magnetic induction at a point 0.2 m from the wire.
2. An infinitely long conductor carries a current of 10 mA. Find the magnetic field.
3. At what distance from a long straight wire carrying a current of 12 A will the magnetic induction be equal to 3×10^5 T?
4. A long straight wire carries a current of 10 A. An electron travels with a velocity of 5×10^6 m/s parallel to the wire 0.1 m from it, and in a direction opposite to the current. What force does the magnetic field of current exert on the electron?
5. A current of 1 amp is flowing in a circular coil of radius 10 cm and 20 turns. Calculate the magnetic field at a distance 10 cm on the axis of the coil and at the centre.
6. Calculate the intensity of magnetic field at the centre of a circular coil of radius 20 cm and 40 turns having a current 2A in it.
7. A current of 1 amp is flowing in a circular coil of radius 10 cm and 20 turns. Calculate the intensity of magnetic field at a distance 10 cm on the axis of the coil and the centre.
8. A solenoid of length 100 cm has 1000 turns wound on it. Calculate the magnetic field at the middle point of its axis when a current of 2A is passed through it.
9. A solenoid of length 20 cm and radius 2cm is closely wound with 200 turns. Calculate the magnetic field intensity at either end of solenoid when the current in the windings is 5 amp.
10. A long solenoid has 20 turns per cm. calculate the magnetic induction at the interior point on the axis for a current of 20 mA.
11. The single carrier holes in a shaped silicon sample is 2.05×10^{22} m^{-3}. Calculate its Hall coefficient. Electron charge = 1.602×10^{-19} C.
12. A copper strip 2 cm wide and 1 mm thick is placed in a magnetic field with B = 1.5 W/m^2 with its thickness parallel to B. If a current of 200 amp is setup in the strip, what Hall potential is developed across the strip? The number of conduction electrons in the copper strip is 8.4×10^{28} $/m^3$.

Chapter – 4: Electromagnetic induction

- **Introduction**:

In 1831, Faraday discovered that "whenever magnetic lines of force are cut by a closed circuit, an induced current flows in the circuit. The e.m.f giving rise to such currents is called induced electromotive force and the phenomenon is called electromagnetic induction.

- **Faraday's experiment to demonstrate electromagnetic induction**:

Experiment 1:

Consider a coil of wire connected in series with a galvanometer (G). When the magnet is inserted in the coil, the galvanometer shows a deflection in one direction and when it is withdrawn from the coil, the galvanometer shows the deflection in the opposite direction. This indicates a momentary current in the coil. When the magnet is stationary, there is no deflection in the galvanometer. If the experiment is repeated with magnetic poles reversed, deflections are also reversed. It is also observed that when the magnet is moved fast, the deflection in the galvanometer is large and when it is moved slowly, the deflection is small i.e., the deflection depends upon the rate at which magnet is inserted or withdrawn.

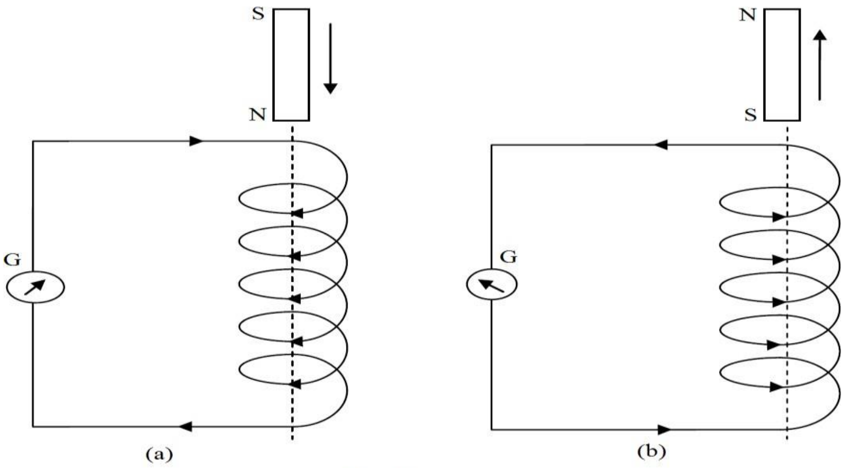


Fig 4.1

Experiment 2:

Consider a primary coil P connected to a battery and secondary coil S connected to a galvanometer. When the circuit is closed by pressing the key K, the galvanometer shows the deflection in one direction. When the circuit is open galvanometer shows the deflection in opposite direction. It is also observed that no deflection is produced in the galvanometer when current flows steadily in the circuit.

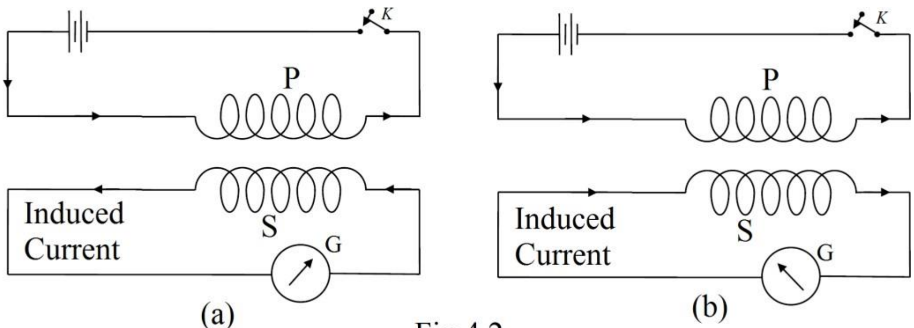


Fig 4.2

Similar effects are observed while increasing or decreasing the primary current or changing the relative position of the coils.

❖ **Explanation of induced e.m.f**:

Consider a magnet and a coil experiment. When the magnet is moved towards the coil, the flux through the coil increases. When the magnet is moved away from the coil, the flux through the coil decreases. In both the cases an induced e.m.f is obtained in the coil during the motion of the magnet.

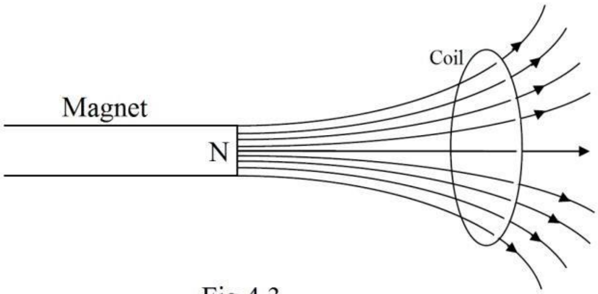


Fig 4.3

❖ **Faraday's laws of electromagnetic induction**:

1st Law: When the magnetic flux linked with a circuit is changed, an e.m.f is induced in the circuit.

2nd Law: The magnitude of the induced e.m.f is directly proportional to the negative rate of change of magnetic flux linked with the circuit.

$e = -\frac{d\Phi_B}{dt}$ .... (1)

Where Φ_B - Magnetic flux

e − Induced e.m.f

Here ′ − ′ sign denotes Lenz law

Faraday's second law is also known as Neumann's law.

Integral and differential form of Faraday's law:

Consider the magnetic field produced by a closed circuit C of any shape which encloses a surface S in the field as shown in fig.

The magnetic flux through a small area dS is $d\Phi_B = B \cdot dS$

The flux through the entire circuit is $\Phi_B = \int_S B \cdot dS$ .... (2)

When the magnetic flux is changed, an electric field is induced around the circuit. The line integral of the electric field gives the induced e.m.f in the closed circuit.

$e = E \cdot dl$ .... (3)

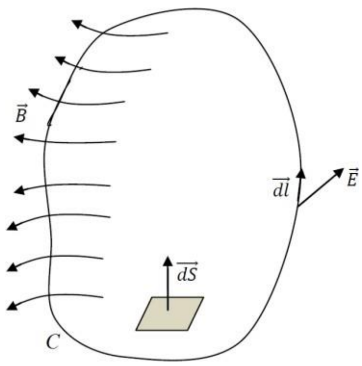


Fig 4.4

Substitute the values of e and Φ_B from equations (3) and (2) in equation (1),

$$E \cdot dl = -\frac{d}{dt}\int_S B \cdot dS \quad \text{............. (4)}$$

This is the integral form of Faraday's law.

From Stokes theorem, $E \cdot dl = \int_S \nabla \times E \cdot dS$ ——————— (5)

From equations (4) and (5),

$$\int_S \nabla \times E \cdot dS = -\frac{d}{dt}\int_S B \cdot dS$$

$$\int_S (\nabla \times E) \cdot dS = -\int_S \frac{\partial B}{\partial t} \cdot dS$$

Comparing on both sides,

$\nabla \times E = -\frac{\partial B}{\partial t}$ or curl $E = -\frac{\partial B}{\partial t}$

This is the differential form of Faraday's law.

- **Lenz's law**:

According to Lenz's law, *the direction of induced e.m.f (or current) in a closed circuit is such that it opposes the original cause that produces it*. This law is based on the principle of law of conservation of energy.

So, when the magnetic flux linked with a circuit increases, the induced e.m.f developed such that it opposes the increase and vice versa.

Explanation:

Suppose the north pole of a magnet is moved towards a coil. As the magnet is pushed towards the coil, an induced e.m.f is setup in the coil. Due to this current the coil behaves as a magnet. The face of the coil towards the north pole of the becomes North Pole and hence the motion of the magnet is opposed.

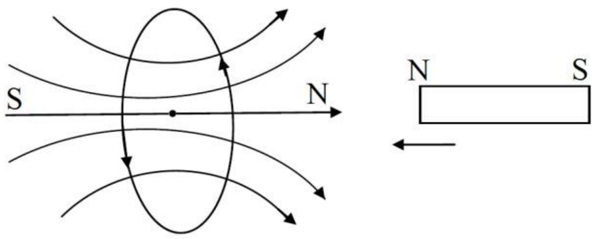


Fig 4.5

Similarly if magnet is moved away from the coil then also the current developed in the circuit opposes the motion of the magnet. So the direction of induced e.m.f always opposes the cause that produced it.

Lenz's law – consequence of conservation of energy:

Assume that due to the motion of the magnet current is developed as shown in the fig. The face of the coil towards the magnet become south. Hence the coil attract magnet and kinetic energy of magnet increases continuously. This is contradiction to the law of conservation of energy. So, current never produces as shown in fig. Hence, we say that Lenz's law is in accordance with the law of conservation of energy.

- **Self induction**:

When a current increases or decreases through a coil, then the coil opposes the change in the current by producing a back e.m.f. This phenomenon is called self induction.

"The property of circuit or coil by virtue of which any change in the magnetic flux linked with it, induces an e.m.f in it, is called self inductance and the induced e.m.f is called back e.m.f."

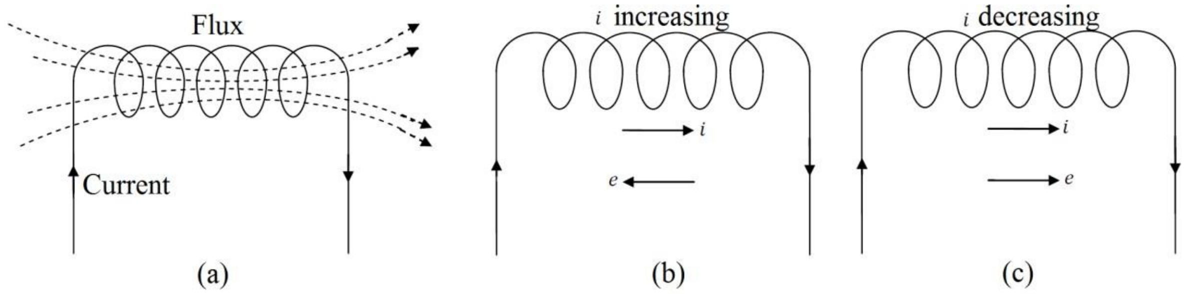


Fig 4.6

When the current is switched on, self induction opposes the growth of current, and when current is switched off, the self induction opposes the decay of current.

Coefficient of self induction:

The total magnetic flux Φ_B linked with a coil is proportional to the current i, flowing in it, i.e., $\Phi_B \propto i$

$\Phi_B = Li$

L − self inductance of the coil.

Definition 1: when $i = 1$ then $\Phi_B = L$. Hence the coefficient of self induction is numerically equal to the magnetic flux linked with coil when unit current flows through it.

Definition 2: we know that $e = -\frac{d\Phi_B}{dt}$

$$e = -\frac{d\ Li}{dt}$$

$$= -L\frac{di}{dt}$$

When $\frac{di}{dt} = 1$, $e = -L$

The coefficient of self inductance is numerically equal to the induced e.m.f in the coil, when the rate of change of current is unity.

Unit: The unit of self inductance is henry which is the inductance of a coil in which an e.m.f of 1 volt is setup by a change of current at 1 ampere per second.

❖ **Self inductance of a long solenoid**:

Consider a long air core solenoid (of small diameter) of length l meter and uniform cross sectional area A metre^2. Let n be the number of turns per metre. Suppose a current i flows through it.

The magnetic field inside the solenoid us given by $B = \mu_0 ni$ weber/m^2

∴ Magnetic flux through each turn $\Phi_B = BA = \mu_0 niA$ Magnetic flux linked with all the turns of solenoid = $\mu_0 niA \times N$ Where N is equal to the total no of turns in the solenoid.

$\Phi_B = \mu_0 niA \times nl$ (∵ $N = nl$)

$= \mu_0 n^2 iAl$

The self inductance of the solenoid $Li = \mu_0 n^2 iAl$

$$\boxed{L = \mu_0 n^2 Al} \text{ henry}$$

where n is the number of turns per unit length.

In terms of total number of turns N of the solenoid $L = \mu_0 \left(\frac{N}{l}\right)^2 Al$

$$L = \frac{\mu_0 N^2 A}{l} \quad \text{henry}$$

❖ **Energy stored in magnetic field or energy stored in inductor**:

Consider a very long solenoid of length l and cross sectional area A. When the current is switched on, self induction opposes the growth of current, i.e., the current flows against back e.m.f and does work against it.

$$dW = -e i dt$$

$$dW = +Li \frac{di}{dt} dt \qquad \because e = -L\frac{di}{dt}$$

Hence, the total work done in bringing the current from zero to a steady maximum value i_0 is

$$W = \int_0^{i_0} Li \frac{di}{dt} dt = L \int_0^{i_0} i \, di$$

$$W = L\left[\frac{i^2}{2}\right]_0^{i_0}$$

$$W = L\frac{i_0^2}{2}$$

$$W = \frac{1}{2} L i_0^2$$

This work done is stored as magnetic field energy.

$U =$ Energy stored $= \frac{1}{2} L i_0^2$

The inductance of the solenoid is given by $L = \mu_0 n^2 Al$

$$U = \frac{1}{2}\mu_0 n^2 Al \, i_0^2$$

$$U = \frac{1}{2}\frac{(\mu_0 n i_0)^2}{\mu_0} Al$$

The magnetic field inside the solenoid $B = \mu_0 n i_0$

$$U = \frac{1}{2}\frac{B^2}{\mu_0} Al$$

The energy density (energy per unit volume) u is given by

$$u = \frac{U}{Al} = \frac{B^2}{2\mu_0}$$

$$u = \frac{B^2}{2\mu_0} \quad \text{Joule/metre}^3$$

❖ **Mutual induction**:

Consider two coils P & S, when the current in the primary coil P changes, then an e.m.f is induced in the secondary coil S. This phenomenon is known as mutual induction.

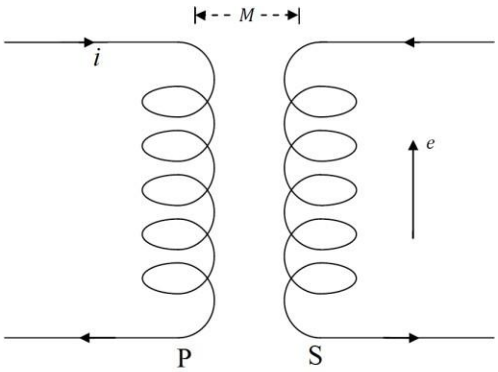


Fig 4.7

We know that flux linked with the secondary is proportional to the current in the primary.

$\Phi_B \propto i$

$\Phi_B = M$

Where M is a constant called the "Coefficient of Mutual induction" or "Mutual inductance" of the two coils.

The e.m.f induced in the secondary S is given by

$$e = -\frac{d\Phi_B}{dt}$$

$$e = -\frac{d}{dt}Mi = -M\frac{di}{dt}$$

Definition 1: when $i = 1$, $\Phi_B = M$, Hence the coefficient of mutual induction is numerically equal to the magnetic flux linked with secondary coil when unit current flows in the primary coil.

Definition 2: when $\frac{di}{dt} = 1$, $e = -M$, "The Mutual inductance M is nothing but the e.m.f induced in the secondary coil, when the rate of change of current is unity in the primary".

- **Coefficient of Coupling (Coupling of two coils with flux linkage)**:

Consider two coils very close to each other and having number of turns N_1 & N_2. Let i_1 , i_2 be the current flowing through the two coils. Now we calculate the mutual inductance of the coils in terms of their self-inductance. By the definition of self inductance

$N_1\Phi_1 = L_1 i_1 \Rightarrow L_1 = \frac{N_1\Phi_1}{i_1}$ .... (1)

$N_2\Phi_2 = L_2 i_2 \Rightarrow L_2 = \frac{N_2\Phi_2}{i_2}$ .... (2)

Here Φ_P and Φ_S are the magnetic fluxes linked with coils 1 and 2 due to their own currents.

$L_1 -$ Self inductance of coil 1

$L_2 -$ Self inductance of coil 2

Let Φ_{21} be the flux linked with each turn of the secondary coil due to the current in the first. Similarly, Φ_{12} be the flux linked with each turn of the first coil due to the current in the second coil.

∴ From the definition of mutual inductance

$N_2\Phi_{21} = M_{21}i_1$ ....(3)

$N_1\Phi_{12} = M_{12}i_2$ ....(4)

If the two coils are wound on the same core so that the centre flux set up by either coil links with all the turns of the other, then the coupling is said to be perfect.

Now $\Phi_{12} = \Phi_2$ and $\Phi_{21} = \Phi_1$ ....(5)

$M_{21} = M_{12} = M_{max}$ .... (6)

From (3) & (4),

$N_2\Phi_1 = M_{max}\, i_1$ .... (7)

$N_1\Phi_2 = M_{max}\, i_2$ .... (8)

Multiplying (7) & (8), $M^2_{max}\, i_1 i_2 = N_1\Phi_1 N_2\Phi_2$

$$M^2_{max} = \frac{N_1\Phi_1}{i_1}\frac{N_2\Phi_2}{i_2}$$

$$M^2_{max} = L_1 L_2$$

$M_{max} = \sqrt{L_1 L_2}$

The above equation is true when the whole of the effective flux from one coil links with the other. In actual practice, the above condition is never fulfilled.

We express the mutual inductance between two coils as $M_{max} = K\sqrt{L_1 L_2}$

Where K is called as coefficient of coupling between the two coils. Its value varies from 0 to 1 and depends upon the geometrical shape of the two coils and their relative positions.

If $K = 1$, the coupling is tight, i.e., no leakage of flux.

If $K = 0$, there is no coupling between the two coils.

If $K > 0$ and $K < 1$, there is optimum coupling.

❖ **<u>Transformer</u>**:

A transformer is an A.C static device which transfers electric power from one circuit to another. It can raise or lower the voltage in a circuit but with a corresponding decrease or increase in current.

Construction:

The transformer consists of two coils. One is known as primary coil P while the other is known as secondary coil S. The two separate coils wound on the same magnetic core but are electrically insulated.

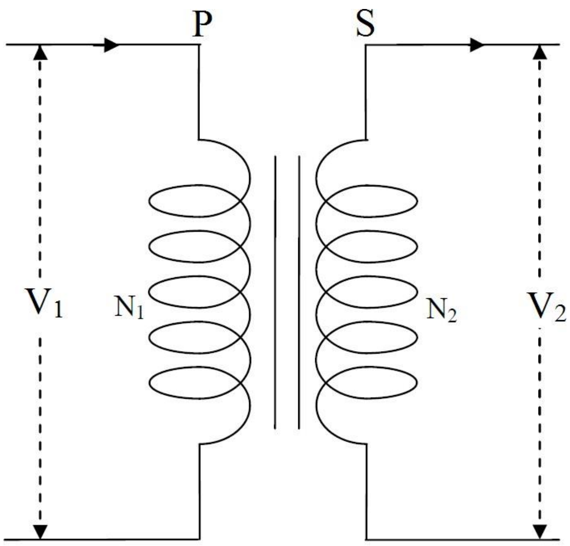


Fig 4.8

The number of turns in the primary coil are N_1

The number of turns in the secondary coil are N_2

Transformer ratio or turns ratio $a = \frac{N_2}{N_1}$

- When the number of turns in the secondary N_2 more than the number of turns in the primary N_1 i.e., $N_2 > N_1$, then the transformer is known as ***step-up transformer.***
- When the number of turns in the primary N_1 more than the number of turns in the secondary N_2 i.e., $N_1 > N_2$, then the transformer is known as ***step-down transformer***.

<u>Principle</u>:

A transformer operates on the principle of mutual induction. When an alternating voltage is applied to the primary, an alternating current is setup in it. It induces a mutually induced e.m.f in the secondary of the same frequency.

Let an a.c. current i_1 flows through the primary coil. This causes a magnetic flux through primary. This induces an e.m.f equal and opposite to V_1.

$V_1 = \varepsilon_1 = -N_1 \frac{d\Phi}{dt}$ .... (1)

The same flux is linked with the secondary coil. Therefore, the secondary voltage is given by

$$V_2 = \varepsilon_2 = -N_2 \frac{d\Phi}{dt} \qquad \text{............... (2)}$$

$$\frac{(2)}{(1)} \Rightarrow \frac{V_2}{V_1} = \frac{N_2}{N_1} = a \text{ or } \frac{V_1}{N_1} = \frac{V_2}{N_2}$$

For step-up transformer, $a > 1$

$V_2 > V_1$ i.e., A step-up transformer raise the voltage.

From law of conservation of energy,

Input power = Output power

$$V_1 i_1 = V_2 i_2$$

$$\frac{V_2}{V_1} = \frac{i_1}{i_2} \Rightarrow V \propto \frac{1}{i}$$

So, In a transformer whatever we gain in voltage, we lose of it in current & vice versa.

Efficiency:

The efficiency of a transformer is defined as the ratio of output power to the input power.

$$\eta = \frac{\text{Output power}}{\text{Input power}} = \frac{V_2 i_2}{V_1 i_1}$$

Applications:

- For long distance transmission of electricity.
- In the manufacture of radio transmitters, tape recorders, etc.
- In welding.
- In rectification of a.c into d.c
- Radio communication, electronic circuits, etc.

❖ **<u>Power (Energy) losses in a transformer</u>**:

In practical usage, some energy losses occur inside a transformer, resulting in less power output than the input power.

1. Finite resistance of the winding
2. Core losses
3. Magnetic leakage.

1. **Finite resistance of the winding**:

   Due to finite resistance of the winding, heat is developed in primary and secondary coils. i^2R heat contributes to power loss in a transformer. The losses are also called as copper losses.

2. **Core losses**:

   The core losses are (a) Iron losses (b) Hysteresis losses.

   (a) **Iron losses**: induced currents and induced em.f are produced inside the core of the transformer. As a result, a part of the energy is wasted due to the generation of eddy currents.

   (b) **Hysteresis losses**: we know that some energy is required to magnetize the core. When secondary is closed, some energy is lost in magnetizing the core. When secondary is open, a very little current in primary, magnetizes the core. These are called hysteresis losses.

3. **Magnetic leakage losses**:

   We assumed that the entire magnetic flux of the primary links with the secondary also, but in practise there will be some leakage of flux. This result in a loss of energy supplied to the primary. This energy lost is known as magnetic leakage loss.

❖ **Important Questions**:

1. State and explain Faraday's law.
2. State and explain Lenz's law.
3. Define coefficient of self inductance.
4. Define coefficient of mutual inductance.
5. Obtain an expression for the energy stored in a solenoid.
6. Derive an expression for the coefficient of coupling in the case of pair of coils.
7. Define coefficient of the self induction and obtain an expression for self inductance of a solenoid.
8. Explain the terms of self inductance and mutual inductance. Prove that $M = \sqrt{L_1 L_2}$
9. Explain the construction and working of transformer. Write its applications.
10. Describe the construction and working of transformer. Explain its energy losses and efficiency.

Problems:

1. A coil of 5 turns has dimensions 9 cm × 7 cm. It rotates at the rate of 15π rad/sec in a uniform magnetic field whose flux density is 0.8 weber/metre^2.
2. A coil of 160 turns of cross-sectional area 250 cm^2 rotates at an angular velocity of 300 rad/ sec about an axis parallel to the plane of the coil in a uniform magnetic field of 0.6 weber/metre^2. What is the maximum e.m.f induced in the coil?
3. Calculate the self inductance of a solenoid of length 1 metre and area of cross-section 0.01 m^2 with 200 turns.
4. What is the self inductance of a 50 cm long solenoid with 2 cm diameter and having 200 turns? $\mu_0 = 4\pi \times 10^{-7} H/m$
5. A solenoid of length 0.50m wound with 5000turns/m of wire has a radius 4 cm. Calculate the self inductance of solenoid.
6. A solenoid with self inductance of 100mH has 500 turns in it. If the number of turns are doubled, what is the self inductance?
7. A coil has 600 turns. Its self inductance is 100mH. Find the self inductance of another same type of coil having 500 turns.
8. A coil has an inductance 50 mH and 100 turns. Calculate the flux linked with it when 20 × 10^{-3} A current is passed through it.

9. The current in the primary circuit of coils changes from 10 amp to 0 in a time of 0.1s. Find the induced e.m.f in the secondary coil. The mutual inductance between the two coils is given to be 2H.
10. In a spark coil *emf* of 40,000 V is induced in the secondary when the primary current changes from 4 amp to 0 amp in 1.0 μs. Find the mutual inductance between the primary and secondary windings to this coil.
11. Calculate the mutual inductance between two coils when a current of 4 amp. Changes to 12 amp. in 0.50 sec and induces an e.m.f of 50 mV in the secondary.
12. Calculate the energy stored in the magnetic field of a solenoid of inductance 5 mH, when a maximum current of 3 amp flows through it.
13. A coil of 200 turns carrying a current of 10 amp produces a magnetic flux of 10 weber/turn. Calculate the energy stored in a magnetic field.
14. A transformer converts 100 V A.C into 1000 V A.C. Find the ratio of number of turns of the primary to the secondary.
15. In a transformer, there are 200 turns in primary coil and 400 turns in secondary coil. If the current in primary coil is 2 amp. Find current in secondary coil.

Unit – III

Chapter – 5: Alternating currents & Electromagnetic waves

❖ **Alternating current**:

An alternating current or a.c is defined as one which passes through a cycle of changes at regular intervals. The waveform of such a voltage or current is shown in fig. and is mathematically represented by

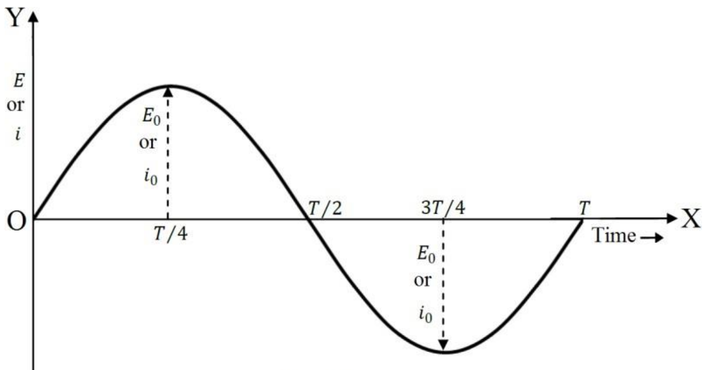


Fig 5.1

$i = i_0 \sin \omega t$

$E = E_0 \sin \omega t$

Here i or E represents the instantaneous value whereas the peak or maximum value is represented by i_0 and E_0. The term ωt is called the phase.

The time taken for one cycle is known as time period represented by T and the number of cycles per second gives the frequency of supply ($f = 1/T$)

❖ **Average value of a.c during one complete cycle**:

The average or mean value of a.c in one complete cycle is zero. The value of current at any instant t is given by $i = i_0 \sin \omega t$.

The average value of a sinusoidal wave over one complete cycle is given by

$$i_{av} = \frac{\int_0^T i_0 \sin \omega t \, dt}{\int_0^T dt} = -\frac{\frac{i_0}{\omega}\left[\cos \omega t\right]_0^T}{T}$$

$$= -\frac{i_0}{\omega T}\left[\cos \frac{2\pi}{T} t\right]_0^T \quad \text{since } \omega = \frac{2\pi}{T}$$

$$= -\frac{i_0}{\omega T}\left[\cos 2\pi - \cos 0\right] = -\frac{i_0}{\omega T}\left[1 - 1\right] = 0$$

Thus, ***the average value of a.c over one complete cycle is zero***.

❖ **Average value of a.c during half cycle**:

The mean or average of a.c is the average of the sum of the instantaneous values taken for half a cycle.

The sum of the instantaneous values for half a cycle is given by

$$= \int_0^{T/2} i\,dt = \int_0^{T/2} i_0 \sin \omega t \, dt$$

$$\therefore \text{Mean value of a.c} = \frac{\int_0^{T/2} i_0 \sin \omega t \, dt}{\int_0^{T/2} dt} = \frac{2}{T}\int_0^{T/2} i_0 \sin \omega t \, dt$$

$$= \frac{2i_0}{T}\left[-\frac{\cos \omega t}{\omega}\right]_0^{T/2} = \frac{2i_0}{T} \times \frac{T}{2\pi}\left[-\cos \frac{2\pi}{T} t\right]_0^{T/2}$$

$$= \frac{i_0}{\pi}\left[-\cos \pi + \cos 0\right]$$

Mean value $i_{av} = \frac{2i_0}{\pi}$

Note: Similarly, for $E = E_0 \sin \omega t$

The average value of alternating voltage for complete cycle is zero.

The average value of alternating voltage for half cycle is $\frac{2i_0}{\pi}$.

❖ **R.M.S value**:

The R.M.S value is the square root of the average of the sum of the square of the instantaneous values taken for one cycle.

R.M.S value of current:

The sum of the squares of the instantaneous values for a period is given by $\int_0^T i_0^2 \sin \omega t \, dt$

Mean square value $= \frac{\int_0^T i_0^2 \sin^2 \omega t \, dt}{\int_0^T dt}$

$= \frac{1}{T}\int_0^T i_0^2 \sin^2 \omega t \, dt$

The value of the integral $\int_0^T \sin^2 \omega t \, dt$ is given by

$\int_0^T \sin^2 \omega t \, dt = \frac{1}{2}\int_0^T (1 - \cos 2\omega t) \, dt$

$= \frac{1}{2}\left[\int_0^T dt - \int_0^T \cos 2\omega t \, dt\right]$

$= \frac{1}{2}\left[T - \left(\frac{\sin 2\omega t}{2\omega}\right)_0^T\right] = \frac{T}{2}$

Mean square value $= \frac{1}{T} \times i_0^2 \times \frac{T}{2} = \frac{i_0^2}{2}$

∴R.M.S value of current $i_{rms} = \frac{i_0}{\sqrt{2}}$

R.M.S value of voltage:

The sum of the squares of the instantaneous values for a period is given by $\int_0^T E_0^2 \sin \omega t \, dt$

Mean square value $= \frac{\int_0^T E_0^2 \sin^2 \omega t \, dt}{\int_0^T dt}$

$= \frac{1}{T}\int_0^T E_0^2 \sin^2 \omega t \, dt$

The value of the integral $\int_0^T \sin^2 \omega t \, dt$ is given by

$\int_0^T \sin^2 \omega t \, dt = \frac{1}{2}\int_0^T (1 - \cos 2\omega t) \, dt$

$= \frac{1}{2}\left[\int_0^T dt - \int_0^T \cos 2\omega t \, dt\right]$

$= \frac{1}{2}\left[T - \left(\frac{\sin 2\omega t}{2\omega}\right)_0^T\right] = \frac{T}{2}$

Mean square value $= \frac{1}{T} \times E_0^2 \times \frac{T}{2} = \frac{E_0^2}{2}$

∴ R.M.S value of voltage $= \frac{E_0}{\sqrt{2}}$

❖ **Form factor**:

It is defined as the ratio of rms value to the average value i.e.,

Form factor $= \frac{i_{rms}}{i_{av}}$

$i_{rms} = \frac{i_0}{\sqrt{2}}$ and $i_{av} = \frac{2i_0}{\pi}$

Form factor $= \frac{i_0}{\sqrt{2}} \times \frac{\pi}{2i_0} = \frac{\pi}{2\sqrt{2}} = 1.11$

❖ **A.C through pure resistance only**:

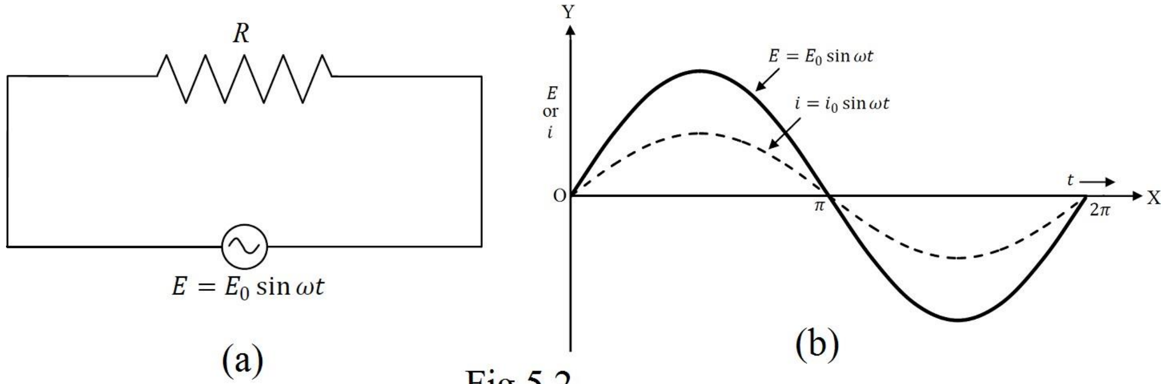


Fig 5.2

Consider a circuit containing pure resistance in series with a.c voltage source. The applied voltage is $E = E_0 \sin \omega t$ .... (1)

By Ohm's law $E = iR$

$E_0 \sin \omega t = iR$

i will be maximum when the term $\sin \omega t$ is unity.

$i = \frac{E_0}{R} \sin \omega t$ ....(2)

$i_0 = \frac{E_0}{R}$ .... (3)

Hence, (2) becomes $i = i_0 \sin \omega t$

From (1) & (4), we conclude that voltage and current are in phase with each other.

❖ **A.C through pure inductance only**:

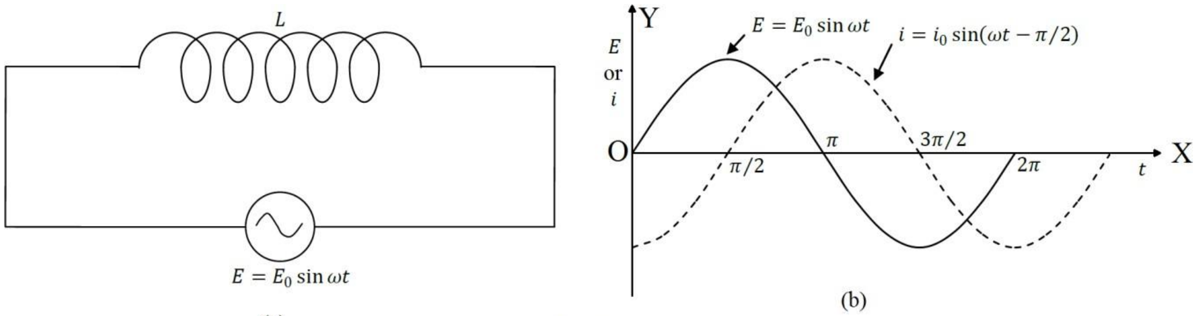


Fig 5.3

Consider a circuit containing pure inductance in series with a.c voltage source. The applied voltage is $E = E_0 \sin \omega t$. Due to the self inductance of the coil an e.m.f is generated which opposes the rise or fall of current through it.

$E = L\frac{di}{dt}$

$E_0 \sin \omega t = L\frac{di}{dt}$

$di = \frac{E_0}{L} \sin \omega t\, dt$ ....(1)

Integrating, we get $i = \frac{E_0}{L}\int \sin \omega t\, dt = -\frac{E_0}{\omega L}\cos \omega t = \frac{E_0}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right)$ ....(2)

Maximum value of i is $i_0 = \frac{E_0}{\omega L}$ when $\sin\left(\omega t - \frac{\pi}{2}\right) = 1$

$i = i_0 \sin\left(\omega t - \frac{\pi}{2}\right)$

The current lags behind the voltage by $\frac{\pi}{2}$ radians or 90^0.

$i_0 = \frac{E_0}{\omega L}$, here ωL plays the role of effective resistance. It is called inductive reactance and is denoted by $X_L = \omega L$.

❖ A.C through pure capacitance only:

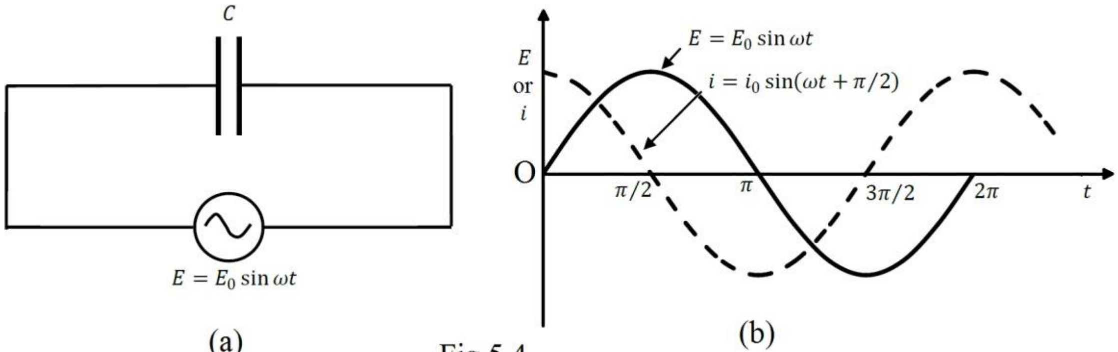


Fig 5.4

Consider a circuit containing pure capacitance only in series with a.c voltage source. The applied voltage is $E = E_0 \sin\omega t$. If q is the charge on plates at any instant, then

$q = CE$ .... (1)

$q = CE_0 \sin\omega t$

From definition of current $i = \frac{dq}{dt} = \frac{d}{dt}\left(CE_0 \sin\omega t\right) = \omega C E_0 \cos\omega t$

$i = \frac{E_0}{1/C\omega}\cos\omega t = \frac{E_0}{1/C\omega}\sin\left(\omega t + \frac{\pi}{2}\right)$

Maximum current $i_0 = \frac{E_0}{1/C\omega}$ when $\sin\left(\omega t + \frac{\pi}{2}\right) = 1$

$i = i_0 \sin\left(\omega t + \frac{\pi}{2}\right)$

The current leads the voltage by $\frac{\pi}{2}$ radians or 90^0.

$i_0 = \frac{E_0}{1/C\omega}$, here $\frac{1}{\omega C}$ plays the role of effective resistance. It is called capacitive reactance and is denoted by $X_C = \frac{1}{\omega C}$

❖ A.C circuit containing resistance and inductance (RL or LR circuit):

Consider a circuit containing resistance R and an inductance L in series connected to a source of alternating voltage as shown in fig. Let i be the current and an induced e.m.f is setup in the inductance which opposes the applied e.m.f and is given by $-L\frac{di}{dt}$.

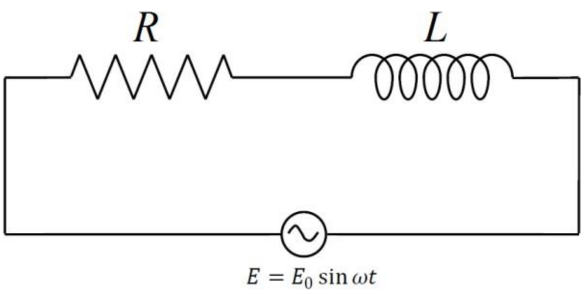


Fig 5.5

According to Ohm's law, $E = L\frac{di}{dt} + Ri$

$L\frac{di}{dt} + Ri = E_0 \sin\omega t$ .... (1)

The above equation is of the form $\frac{dy}{dx} + Py = Q$

The trial solution of equation (1) is $i = i_0 \sin(\omega t - \phi)$ ....(2)

Where i_0 and ϕ are constants.

Differentiate equation (2) with respect to time

$\frac{di}{dt} = i_0 \omega \cos(\omega t - \phi)$.. (3)

Substituting the values of i and $\frac{di}{dt}$ from eqs. (2) and (3) in eq. (1)

$L i_0 \omega \cos(\omega t - \phi) + R i_0 \sin(\omega t - \phi) = E_0 \sin \omega t$

$L i_0 \omega \cos(\omega t - \phi) + R i_0 \sin(\omega t - \phi) = E_0 \sin[(\omega t - \phi) + \phi]$

$L i_0 \omega \cos(\omega t - \phi) + R i_0 \sin(\omega t - \phi) = E_0 \sin(\omega t - \phi) \cos \phi + E_0 \cos(\omega t - \phi) \sin \phi$

Equating the $\cos(\omega t - \phi)$ and $\sin(\omega t - \phi)$ on both sides of above equation

$L i_0 \omega = E_0 \sin \phi$ ....(4)

$R i_0 = E_0 \cos \phi$ ....(5)

Squaring and adding equations (4) and (5),

$E_0^2 = L^2 i_0^2 \omega^2 + R^2 i_0^2 = i_0^2 (R^2 + L^2 \omega^2)$

$\therefore i_0 = \frac{E_0}{\sqrt{R^2 + L^2\omega^2}}$ .... (6)

$\frac{eq\ 4}{eq\ (5)} \Rightarrow \tan \phi = \frac{\omega L}{R}$ .... (7)

From (2), $i = \frac{E_0}{\sqrt{R^2 + L^2\omega^2}} \sin(\omega t - \phi)$ .... (8)

and $\phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$

Equation (8) represents the current in the circuit at any instant.

(i) The amplitude of the current $i_0 = \frac{E_0}{\sqrt{R^2 + L^2\omega^2}}$

(ii) The current lags in phase behind the e.m.f by an angle given by

$\phi = \tan^{-1}\left(\frac{\omega L}{R}\right) = \tan^{-1}\left(\frac{X_L}{R}\right)$

(iii) The impedance Z of the circuit $Z = \frac{E_0}{i_0} = \sqrt{R^2 + L^2\omega^2}$

$Z = \sqrt{R^2 + X_L^2}$

<u>Vector diagram of RL or LR circuit</u>:

Let E_R and E_L be the magnitudes of voltages across the resistance R and inductance L respectively. Current i is same in R and L, So

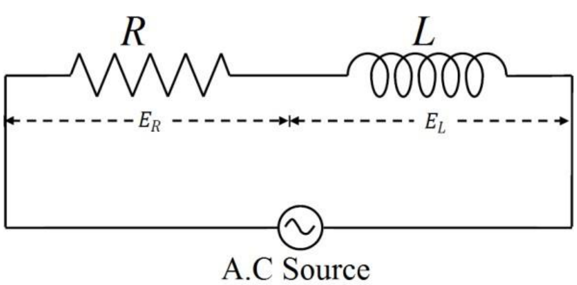


Fig 5.6

$E_R = iR$

$E_L = iX_L = i\omega L$

The vector diagram can be plotted by considering the fact that the voltage across resistance always remain in phase with the current but the voltage across inductance lead over current 90^0. So E_R is represented along the x – axis and E_L at 90^0 ahead to E_R on the y – axis. Thus, E_R and E_L are mutually at right angles as shown in fig.

The resultant voltage across the inductance and resistance in series is given by parallelogram law of vector addition.

$E^2 = E_R^2 + E_L^2$

$(iZ)^2 = (i\omega L)^2 + (iR)^2$

$E = iZ$, where Z is impedance of the circuit.

$i^2Z^2 = i^2\omega^2L^2 + i^2R^2$

$Z^2 = R^2 + \omega^2L^2$

$Z = \sqrt{R^2 + \omega^2L^2}$

$\tan\phi = \frac{E_L}{E_R} = \frac{\omega L}{R}$ or $\phi = \tan^{-1}\frac{\omega L}{R}$

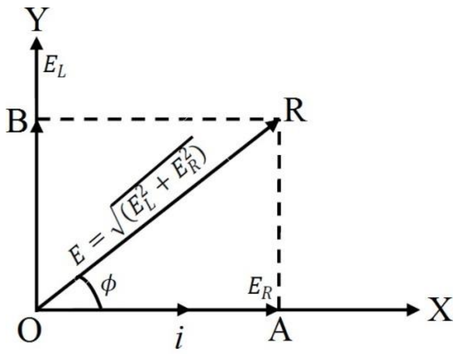


Fig 5.7

- **<u>A.C circuit containing resistance and capacitance (RC or CR circuit)</u>**:

Consider a circuit containing resistance R and a capacitance C in series connected to a source of alternating voltage as shown in fig. Let q be the charge on the capacitor at any instant t and i be the current in the circuit. The potential difference across the capacitor is $\frac{q}{C}$.

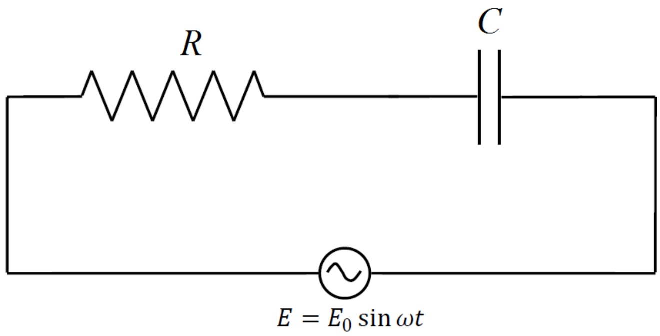


Fig 5.8

According to Ohm's law, $E = \frac{q}{C} + iR$

$\frac{q}{C} + iR = E_0 \sin\omega t$

Differentiating above with respect to time.

$\frac{1}{C}\frac{dq}{dt} + R\frac{di}{dt} = E_0\,\omega\cos\omega t$

$R\frac{di}{dt} + \frac{i}{C} = E_0\,\omega\cos\omega t \qquad \because i = \frac{dq}{dt}$ .... (1)

The above equation is of the form $\frac{dy}{dx} + Py = Q$

The trial solution of equation (1) is $i = i_0 \sin(\omega t - \phi)$ ....(2)

Where i_0 and ϕ are constants.

Differentiate equation (2) with respect to time

$\frac{di}{dt} = i_0\,\omega\cos(\omega t - \phi)$.. (3)

Substituting the values of i and $\frac{di}{dt}$ from eqs. (2) and (3) in eq. (1)

$Ri_0\,\omega\cos(\omega t - \phi) + \frac{i_0}{C}\sin(\omega t - \phi) = E_0\,\omega\cos\omega t$

$Ri_0\,\omega\cos(\omega t - \phi) + \frac{i_0}{C}\sin(\omega t - \phi) = E_0\,\omega\cos(\omega t - \phi + \phi)$

$Ri_0\,\omega\cos(\omega t - \phi) + \frac{i_0}{C}\sin(\omega t - \phi) = E_0\,\omega\sin(\omega t - \phi)\cos\phi + E_0\,\omega\cos(\omega t - \phi)\sin\phi$

Equating the $\cos(\omega t - \phi)$ and $\sin(\omega t - \phi)$ on both sides of above equation

$Ri_0\omega = E_0\omega\sin\phi \Rightarrow Ri_0 = E_0\sin\phi$ ....(4)

$\frac{i_0}{C} = E_0\,\omega\cos\phi \Rightarrow \frac{i_0}{C\omega} = E_0\cos\phi$ ....(5)

Squaring and adding equations (4) and (5),

$$E_0^2 = (Ri_0)^2 + \left(\frac{i_0}{C\omega}\right)^2$$

$$i_0^2 = \frac{E_0^2}{R^2 + \frac{1}{C^2\omega^2}}$$

$$i_0 = \frac{E_0}{\sqrt{R^2 + \frac{1}{C^2\omega^2}}} \quad \text{............ (6)}$$

$$\frac{eq\,4}{eq\,(5)} \Rightarrow \tan\phi = \frac{1/C\omega}{R} \quad \text{............ (7)}$$

From (2), $i = \frac{E_0}{\sqrt{R^2 + \frac{1}{C^2\omega^2}}} \sin(\omega t - \phi)$ .... (8)

and $\phi = \tan^{-1}\left(\frac{1/C\omega}{R}\right)$

Equation (8) represents the current in the circuit at any instant.

(i) The amplitude of the current $i_0 = \frac{E_0}{\sqrt{R^2 + \frac{1}{C^2\omega^2}}}$

(ii) The voltage lags in phase behind the current by an angle given by

$$\phi = \tan^{-1}\left(\frac{1/C\omega}{R}\right) = \tan^{-1}\left(\frac{X_C}{R}\right)$$

(iii) The impedance Z of the circuit $Z = \frac{E_0}{i_0} = \sqrt{R^2 + \frac{1}{C^2\omega^2}}$

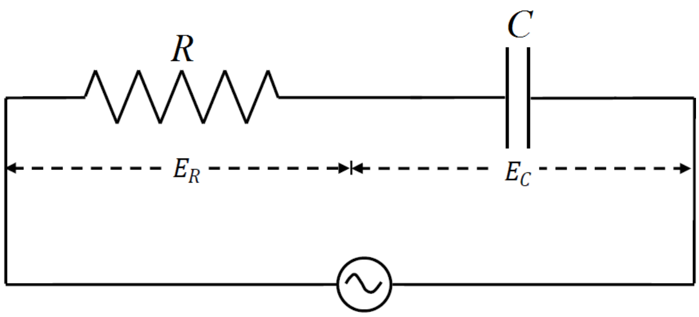
$$Z = \sqrt{R^2 + X_C^2}$$

<u>Vector diagram of RC or CR circuit</u>:

Let E_R and E_C be the magnitudes of voltages across the resistance R and capacitance C respectively. Current i is same in R and C, So

$E_R = iR$

$E_C = iX_C = \frac{i}{\omega C}$



A.C. Source

Fig 5.9

The vector diagram can be plotted by considering the fact that the voltage across resistance always remain in phase with the current but the voltage across capacitance lags behind current 90^0. So E_R is represented along the x – axis and E_C at 90^0 below to E_R on the y – axis. Thus, E_R and E_C are mutually at right angles as shown in fig.

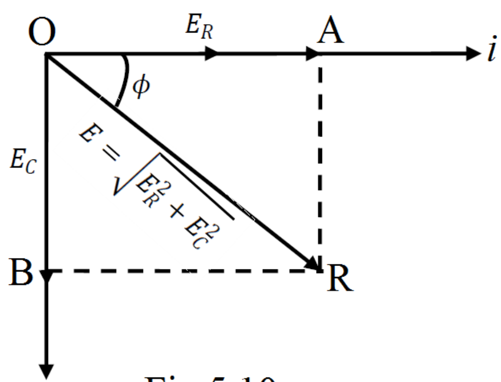


Fig 5.10

The resultant voltage across the capacitance and resistance in series is given by parallelogram law of vector addition.

$$E^2 = E_R^2 + E_C^2$$

$(iZ)^2 = \left(\frac{i}{\omega C}\right)^2 + (iR)^2$

$E = iZ$, where Z is impedance of the circuit.

$i^2Z^2 = \frac{i^2}{\omega^2 C^2} + i^2R^2$

$Z^2 = R^2 + \omega^2C^2$

$Z = \sqrt{R^2 + \omega^2C^2}$

$\tan\phi = \frac{E_C}{E_R} = \frac{1/\omega C}{R}$ or $\phi = \tan^{-1}\left(\frac{1}{\omega CR}\right)$

❖ **<u>A.C circuit containing inductance, capacitance and resistance in series (LCR circuit)</u>**:

Consider a circuit containing resistance R, capacitance C and inductance L in series connected to a source of alternating voltage as shown in fig. Let q be the charge on the capacitor at any instant t and i be the current in the circuit. The potential difference across the capacitor is $\frac{q}{C}$ and the back e.m.f due to self inductance is $L\frac{di}{dt}$.

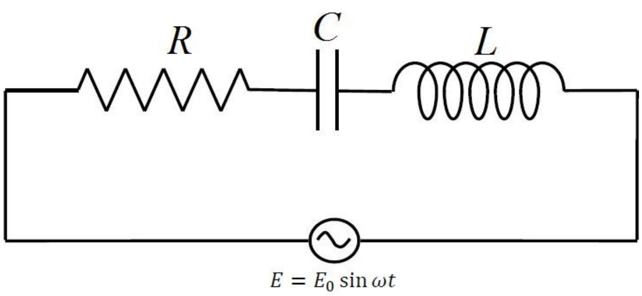


Fig 5.11

According to Ohm's law, $E = L\frac{di}{dt} + iR + \frac{q}{C}$

$L\frac{di}{dt} + iR + \frac{q}{C} = E_0 \sin\omega t$

Differentiating, we get $L\frac{d^2 i}{dt^2} + R\frac{di}{dt} + \frac{1}{C}\frac{dq}{dt} = E_0\,\omega \cos\omega t$

$L\frac{d^2 i}{dt^2} + R\frac{di}{dt} + \frac{i}{C} = E_0\omega \cos\omega t$ .... (1)

The trial solution of equation (1) is $i = i_0 \sin(\omega t - \phi)$ ....(2)

Where i_0 and ϕ are constants.

Differentiate equation (2) with respect to time

$\frac{di}{dt} = i_0\,\omega \cos(\omega t - \phi)$

$\frac{d^2 i}{dt^2} = -i_0\,\omega^2 \sin(\omega t - \phi)$

Substituting the values of i, $\frac{di}{dt}$ and $\frac{d^2 i}{dt^2}$ in equation (1), we get

$-Li_0\omega^2 \sin(\omega t - \phi) + Ri_0\omega \cos(\omega t - \phi) + \frac{1}{C} i_0 \sin(\omega t - \phi) = E_0\omega \cos\omega t$

$-Li_0\omega^2 \sin(\omega t - \phi) + Ri_0\omega \cos(\omega t - \phi) + \frac{1}{C} i_0 \sin(\omega t - \phi) = E_0\omega \cos(\omega t - \phi + \phi)$

$-Li_0\omega^2 \sin(\omega t - \phi) + Ri_0\omega \cos(\omega t - \phi) + \frac{1}{C} i_0 \sin(\omega t - \phi) = E_0\omega \cos(\omega t - \phi)\cos\phi - E_0\omega \sin(\omega t - \phi)\sin\phi$

Comparing the coefficients of $\sin(\omega t - \phi)$ and $\cos(\omega t - \phi)$ on both sides of above eq

$-Li_0\omega^2 + \frac{i_0}{C} = -E_0\,\omega \sin\phi$ or $-L\omega^2 i_0 + \frac{1}{C} i_0 = -E_0\omega \sin\phi$ .... (3)

$R i_0 \omega = E_0 \omega \cos\phi$ ....(4)

Squaring and adding equations (4) & (5),

$$\left(L - \frac{1}{C}\right)^2 i_0^2 + R^2 i_0^2 \omega^2 = E_0^2 \omega^2$$

$$i_0^2\left[R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2\right] = E_0^2$$

$$i_0 = \frac{E_0}{\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}} \quad \text{................ (5)}$$

$$\frac{\text{eq }3}{\text{eq }(4)} \Rightarrow \tan\phi = -\frac{-L + \frac{1}{C}}{R\omega} = \frac{\omega L - 1/C\omega}{R}$$

(i) The maximum current is given by $i_0 = \dfrac{E_0}{\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}}$

(ii) The impedance of the circuit is $Z = \sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2} = \sqrt{R^2 + (X_L - X_C)^2}$

(iii) When $L\omega > \frac{1}{C\omega}$, ϕ is positive, i.e., current lags behind the applied e.m.f.

When $L\omega = \frac{1}{C\omega}$, $\phi = 0$, the current is in phase with e.m.f

When $L\omega < \frac{1}{C\omega}$, ϕ is negative, i.e., current leads the e.m.f

❖ **Series resonant circuit**:

The LCR series circuit has a very large capacitive reactance $(1/\omega C)$ at low frequencies and a very large inductive reactance (ωL) at high frequencies. So at a particular frequency, the total reactance in the circuit is zero $(\omega L = 1/\omega C)$. Under this condition, the resultant impedance of the circuit is minimum.

The particular frequency of A.C at which impedance of a series LCR circuit becomes minimum (or the current becomes maximum), [when $\omega L = 1/\omega C$] is called the resonant frequency and the circuit is called as series resonant circuit.

At resonant frequency $\omega L = 1/\omega C$

$$\omega^2 = \frac{1}{LC}$$

$$\omega = \frac{1}{\sqrt{LC}}$$

The resonant frequency f_0 of the series resonant circuit is given by

$$2\pi f_0 = \frac{1}{\sqrt{LC}}$$

$$\boxed{f_0 = \frac{1}{2\pi\sqrt{LC}}}$$

The variation of the peak value of the current with frequency of the applied e.m.f is shown in fig. we observe the following points.

(i) The maximum current occurs at a particular frequency called as resonant frequency f_0.

(ii) The peak value of the curve depends on the resistance of the circuit. When R is low, the peak value is high and vice – versa. The peak is known as sharpness of resonance.

(iii) The series resonant circuit is sometimes called as ***acceptor circuit***. The reason is that impedance of the circuit is minimum at resonance and due to this fact it readily accept that current out of the many currents whose frequency is equal to its resonant frequency.

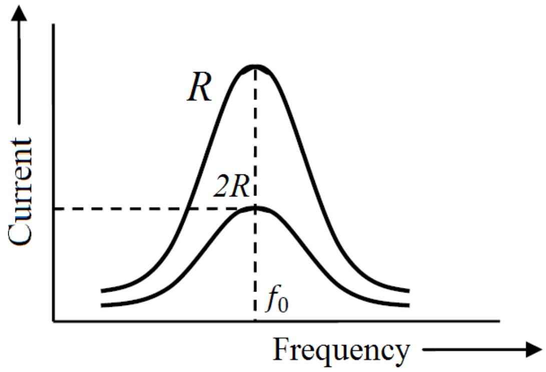


Fig 5.12

❖ **<u>Parallel resonant circuit</u>**:

A parallel resonant circuit is shown in fig. Here an inductance L and a resistance R are connected in series in one branch and a capacitance C in another branch. An A.C voltage source is commonly connected to two branches. The current is divided into two branches at the junction point. From Kirchhoff's law

$i_0 = i_1 + i_2$ .... (1)

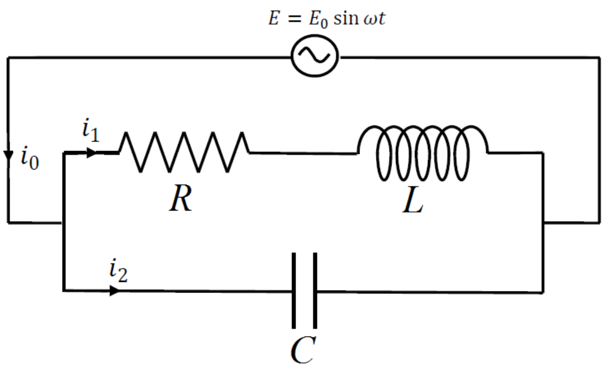


Fig 5.13

Let Z be the impedance of the circuit.

Impedance of inductance and resistance branch, $Z_1 = R + j\omega L$

Impedance of condenser branch, $Z_2 = \frac{1}{j\omega C}$

The resultant impedance for a parallel circuit is given by $\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2}$

$$\frac{1}{Z} = \frac{1}{R+j\omega L} + \frac{1}{1/j\omega C}$$

$$\frac{1}{Z} = \frac{1}{R+j\omega L} + j\omega C$$

Admittance $Y = \frac{1}{Z} = \frac{1}{R+j\omega L} + j\omega C$

$$Y = \frac{R-j\omega L}{(R-j\omega L)(R+j\omega L)} + j\omega C$$

$$= \frac{R-j\omega L}{R^2+\omega^2L^2} + j\omega C$$

$$= \frac{R}{R^2+\omega^2L^2} + j\left(\omega C - \frac{\omega L}{R^2+\omega^2L^2}\right)$$

The magnitude of admittance is given by $Y = \left[\left(\frac{R}{R^2+\omega^2L^2}\right)^2 + \left(\omega C - \frac{\omega L}{R^2+\omega^2L^2}\right)^2\right]^{1/2}$

The admittance is minimum or impedance is maximum at a particular frequency, when

$\omega C = \frac{\omega L}{R^2+\omega^2L^2}$ or $C = \frac{L}{R^2+\omega^2L^2}$

$CR^2 + C\omega^2L^2 = L$

$$C\omega^2L^2 = L - CR^2$$
$$\omega^2 = \frac{L-CR^2}{CL^2} = \frac{1}{LC} - \frac{R^2}{L^2}$$
$$\omega = \left(\frac{1}{LC} - \frac{R^2}{L^2}\right)^{1/2}$$
$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\left(\frac{1}{LC} - \frac{R^2}{L^2}\right)^{1/2}$$

$$f = \frac{1}{2\pi}\left(\frac{1}{LC} - \frac{R^2}{L^2}\right)^{1/2}$$

At this frequency, the admittance is minimum (impedance is maximum) and hence, the current is minimum. Such a frequency is called as ***resonant frequency***. The circuit is known as parallel resonant circuit.

❖ **Quality factor Q of circuit**:

The quality factor is defined as 2π times the ratio of the energy stored to the average energy loss per period. The quality factor is a measure of the efficiency of energy stored in an inductor or capacitor when an alternating current is applied.

$$Q = 2\pi \frac{\text{energy stored}}{\text{energy lost per period}}$$

The energy is stored in form of electric field between the plates of a capacitor. When V_0 is applied voltage across the plates of a capacitor,

Energy stored in capacitor
$$E_C = \frac{1}{2}CV_0^2$$

The energy is stored in the form of magnetic field around the inductor. When i_0 is the current passing through the inductor,

Energy stored in inductor
$$E_L = \frac{1}{2}Li_0^2$$

$$Q = 2\pi \frac{\text{energy stored}}{\text{energy lost per period}}$$
$$Q = 2\pi f \frac{\text{energy stored}}{\text{power lost in one second}}$$
$$Q = 2\pi f \frac{\frac{1}{2}Li_0^2}{\frac{1}{2}i_0^2R}$$
$$Q = \frac{2\pi f L}{R} = \frac{\omega L}{R}$$

So, the quality factor may also be defined as the ratio of reactance of either inductance or capacitance at the resonant frequency to the circuit.

❖ **<u>Series resonance versus parallel resonance</u>:**

Series resonant circuit	**Parallel resonant circuit**
1. Series resonant frequency is given by $f_r = \frac{1}{2\pi\sqrt{LC}}$ independent of resistance.	1. Parallel resonant frequency is given by $f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$
2. At resonance, the power factor is unity and impedance is purely resistive $Z_r = R$	2. Power factor is also unity but the impedance is given by $Z_r = \frac{L}{CR}$
3. At resonance, the current is maximum.	3. At resonance, the current is minimum.
4. At resonance, the impedance of the circuit is minimum.	4. At resonance, the impedance of the circuit is maximum.
5. The circuit is called as acceptor circuit because it accepts a particular frequency and rejects all others.	5. The circuit is called as rejector circuit because it rejects only one frequency and accepts other.
6. At resonance, the circuit exhibits a voltage magnification and it is equal to $Q-$ factor.	6. At resonance, the circuit exhibits current magnification and it is equal to $Q-$ factor.

❖ **Important Questions**:

1. What is RMS value of current and voltage?
2. What is $Q-$ factor? Write expression for $Q-$ factor.
3. Distinguish between series and parallel resonance.
4. Derive an expression for current, impedance and phase angle in LR circuit with the help of phase diagram.
5. Derive an expression for current, impedance and phase angle in CR circuit with the help of phase diagram.
6. Explain LCR series resonant circuit. Why it is called as acceptor circuit?
7. Explain LCR parallel resonance circuit.

❖ **Problems**:

1. An alternating current of 50Hz has maximum value of 100A. Find its value after $\frac{1}{600}$ sec.
2. A.C voltage of 180V with frequency of 50cps is connected in series with a resistance 100Ω and a coil of inductance 0.2H. Calculate the R.M.S value of current.
3. A series circuit of $R = 25\Omega$ and $L = 0.2H$ is to be used at a frequency of 500 Hz. Find the impedance.
4. Calculate the resonant frequency of an LCR parallel resonant circuit with $L = 10$ mH, $C = 1\mu F$ and $R = 1\Omega$.
5. A series LCR resonance circuit of resistance 2000 Ω, capacitance 1250 pF and inductance 20 μH is connected to an alternating source of 1 MHz delivering 10V effective voltage. Calculate the resonant frequency and the maximum current in the circuit.
6. A series resonant circuit is formed with a condenser of capacity 250 pF, a coil of inductance 0.16 mH and a resistance 20Ω. Calculate the frequency of resonance and impedance of resonance.
7. In a series RLC circuit $R = 100\Omega$, $L = 0.5H$ and $C = 40\mu F$. Calculate resonant frequency and $Q-$factor.

Unit - III
Chapter – 6: Maxwell's equations

❖ **Introduction**:

Maxwell in 1862 formulated the basic laws of electricity and magnetism in the form of four fundamental equations. The electromagnetic field is described in terms of these set of four equations which are known as ***Maxwell's equations***.

❖ **Basic laws of electricity and magnetism**:

1. **Gauss's law of electrostatics**:

$$E \cdot dS = \frac{q}{\varepsilon_0}$$

This is Gauss's law of electrostatics which states that the electric flux through a closed surface is equal to the net charge enclosed by the surface divided by the permittivity constant ε_0.

2. **Gauss's law of magnetism**:

$$B \cdot dS = 0$$

This is Gauss's law of magnetism. This states that the magnetic flux through a closed surface is zero.

3. **Faraday's law of electromagnetic induction**:

$$E \cdot dl = -\frac{d\Phi_B}{dt}$$

This is Faraday's law of electromagnetic induction.
This law states that an electric field is produced by changing magnetic field.

4. **Ampere's law**:

$$B \cdot dl = \mu_0 i$$

This is Ampere's law for magnetic field due to steady current. This law states that the amount of work done in carrying a unit magnetic pole around a closed arbitrary path linked with the current is μ_0 times the current i.

❖ **Maxwell's equations (Integral and differential forms)**:

Integral forms:	**Differential forms**:
$E \cdot dS = \frac{q}{\varepsilon_0}$	div $E = \frac{\rho}{\varepsilon_0}$ or $\nabla \cdot D = \rho$
$B \cdot dS = 0$	div $B = 0$ or $\nabla \cdot B = 0$
$E \cdot dl = -\frac{d\Phi_B}{dt}$	curl $E = -\frac{\partial B}{\partial t}$ or $\nabla \times E = -\frac{\partial B}{\partial t}$
$B \cdot dl = \mu_0 J + \varepsilon_0 \frac{\partial E}{\partial t}$	curl $B = \mu_0 J + \varepsilon_0 \frac{\partial E}{\partial t}$ or $\nabla \times B = \mu_0 J + \varepsilon_0 \frac{\partial E}{\partial t}$

❖ **Displacement current**:

A current carrying conductor produces a magnetic field. Maxwell proved that a changing electric field in vacuum or in dielectric also produces a magnetic field. So, a changing electric field is equivalent to a current which flows as long as the electric field is changing and produces the same magnetic effect as an ordinary conduction current. This is known as ***displacement current***.

Ampere's law in vector form is $\nabla \times B = \mu_0 j$, where j is current density.

Apply divergence on both sides $\nabla \cdot \nabla \times B = \nabla \cdot \mu_0 j$

$\mu_0 \nabla \cdot j = \mu_0 \text{ div } j = 0$ $\quad\because \nabla \cdot \nabla \times B = 0$

div $j = 0$, this is contradict with the equation of continuity which states that

$\text{div } j + \frac{\partial \rho}{\partial t} = 0$ where ρ represents charge density.

Maxwell concluded that Ampere's law is incomplete. Ampere's law in the form $\nabla \times B = \mu_0 j$ is valid only for a steady state condition and is insufficient for the case of time varying electric field in which the charge density varies with time, i.e., $\frac{\partial \rho}{\partial t}$ is not zero.

Maxwell suggested that something must be added to j Thus

$\nabla \times B = \mu_0 j$ + Something

In vector form, the Gauss's law is expressed as $\nabla \cdot D = \rho$

Differentiating with respect to time, we get $\nabla \cdot \frac{\partial D}{\partial t} = \frac{\partial \rho}{\partial t}$

Adding $\nabla \cdot j$ on both sides and rearranging

$\nabla \cdot j + \frac{\partial \rho}{\partial t} = \nabla \cdot j + \frac{\partial D}{\partial t} = 0$

$\nabla \cdot j = 0$ For steady currents

$\nabla \cdot \left(j + \frac{\partial D}{\partial t} \right) = 0$ Everywhere

In this way, Maxwell replaced j in ampere's law by $j + \frac{\partial D}{\partial t}$

Thus, Ampere's law becomes $\nabla \times B = \mu_0 \left(j + \frac{\partial D}{\partial t} \right)$

The term $\frac{\partial D}{\partial t}$ is called as displacement current density.

❖ **Maxwell's wave equation or Equation of electromagnetic waves:**

Consider a homogeneous, isotropic dielectric medium in free space. Dielectric offers infinite resistance to the current, and hence its conductivity $j = 0$. Charge density is zero for homogeneous isotropic medium. Hence

$j = 0$, $\rho = 0$, $D = \varepsilon E$ and $B = \mu_0 \mu_r H$

Maxwell's equations for a dielectric become

$\nabla \cdot E = 0$ .... (1)

$\nabla \cdot B = 0$ .... (2)

$\nabla \times E = -\frac{\partial B}{\partial t}$ .... (3)

$\nabla \times B = \mu\varepsilon \frac{\partial E}{\partial t}$ .... (4)

A. Equation of EM waves can be obtained by eliminating E from eqs. (3) and (4). Taking curl of equation (4), we get

$\nabla \times \nabla \times B = \nabla \times \mu\varepsilon \frac{\partial E}{\partial t}$

$$\nabla\times\nabla\times B = \nabla\times\mu\varepsilon\frac{\partial E}{\partial t}$$
$$= \mu\varepsilon\left(\nabla\times\frac{\partial E}{\partial t}\right)$$
$$= \mu\varepsilon\frac{\partial}{\partial t}(\nabla\times E)$$
$$= \mu\varepsilon\frac{\partial}{\partial t}\left(-\frac{\partial B}{\partial t}\right) \qquad \text{[using eq. (3)]}$$
$$\nabla\times\nabla\times B = -\mu\varepsilon\frac{\partial^2 B}{\partial t^2} \qquad \text{.............. (5)}$$
$$\nabla\times\nabla\times B = \nabla(\nabla\cdot B) - \nabla^2 B$$
$$= \nabla(0) - \nabla^2 B$$
$$= -\nabla^2 B \qquad \text{.............. (6)}$$

Substituting the value of $\nabla\times\nabla\times B$ from equation (6) in eq. (5), we get

$$-\nabla^2 B = -\mu\varepsilon\frac{\partial^2 B}{\partial t^2}$$

$$\boxed{\nabla^2 B = \mu\varepsilon\frac{\partial^2 B}{\partial t^2}} \qquad \text{............. (7)}$$

B. Equation of EM waves can be obtained by eliminating B from eqs. (3) and (4).
Taking curl of equation (3), we get

$$\nabla\times\nabla\times E = \nabla\times\left(-\frac{\partial B}{\partial t}\right)$$
$$= -\frac{\partial}{\partial t}(\nabla\times B)$$
$$= -\mu\varepsilon\frac{\partial}{\partial t}\left(\frac{\partial E}{\partial t}\right) \qquad \text{[using eq. (4)]}$$
$$\nabla\times\nabla\times E = -\mu\varepsilon\frac{\partial^2 E}{\partial t^2} \qquad \text{............... (8)}$$
$$\nabla\times\nabla\times E = \nabla(\nabla\cdot E) - \nabla^2 E$$
$$= \nabla(0) - \nabla^2 E$$
$$= -\nabla^2 E \qquad \text{................ (9)}$$

Substituting the value of $\nabla\times\nabla\times E$ from equation (9) in eq. (8), we get

$$-\nabla^2 E = -\mu\varepsilon\frac{\partial^2 E}{\partial t^2}$$

$$\boxed{\nabla^2 E = \mu\varepsilon\frac{\partial^2 E}{\partial t^2}} \qquad \text{............ (10)}$$

Equations (7) and (10) are called wave equations for B and E respectively. These equations have the same general form of the differential equation of wave motion. The general wave equation is represented by

$$\nabla^2 y = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} \qquad \text{............... (11)}$$

Where v is the velocity of the wave.

Comparing equations (10) and (11), we have $\frac{1}{v^2} = \mu\varepsilon$

$$v^2 = \frac{1}{\mu\varepsilon}$$

$v = \frac{1}{\sqrt{\mu\varepsilon}}$

Where μ and ε are permeability and permittivity of the medium.

Therefore, electric vector E and magnetic field vector B are propagating in space according to wave equation with velocity v . These waves are commonly referred as electromagnetic waves.

❖ **Transverse wave nature of electromagnetic wave**:

Consider the case of electromagnetic wave in which the components of vectors E and B vary with one coordinate only (say x) and also with time t, i.e.,

$E = E(x, t)$ and $B = B(x, t)$

But $\nabla \cdot E = 0$ or $\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = 0$

$\therefore \frac{\partial E_x}{\partial x} = 0$ or E_x = constant.....................................(1)

$\nabla \cdot B = 0$ or $\frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} = 0$

$\therefore \frac{\partial B_x}{\partial x} = 0$ or B_x = constant......................................(2)

Eqs. (1) and (2) are obtained on the fact that the derivative of E and B with respect to y and z are zero.

$curl\, E = -\frac{\partial B}{\partial t}$

$$\therefore \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ E_x & E_y & E_z \end{vmatrix} = -\frac{\partial}{\partial t}(B_x i + B_y j + B_z k)$$

$i\left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right) = -i\frac{\partial B_x}{\partial t}$

$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = 0$ (∵ components of E vary only with x)

$\frac{\partial B_x}{\partial t} = 0$ or B_x = constant

Similarly, taking curl $\boldsymbol{B}$, we can show that E_x = constant.

Hence, we conclude that E_x and B_x are constants as regards to time and space. So these components are static components and hence no part of wave motion. Thus,

$E = E_y j + E_z k$

$B = B_y j + B_z k$

$x -$ direction is the propagation of the wave. Both these vectors are perpendicular to the direction of propagation. Hence, Maxwell's electromagnetic waves are transverse in nature.

❖ **Production and detection of electromagnetic waves (Hertz experiment)**:

Hertz demonstrated the production of EM waves experimentally. The experimental setup is shown in the fig (a). It consist A and B are two metal square plates placed at a certain distance. These plates act as the plate of a capacitor. The opposite faces of A and B are connected to highly polished brass spheres C and D through thick wires. The brass spheres are called as buttons or knobs and are separated by a small gap. An induction coil is connected across the wires attached to the plates of the capacitor.

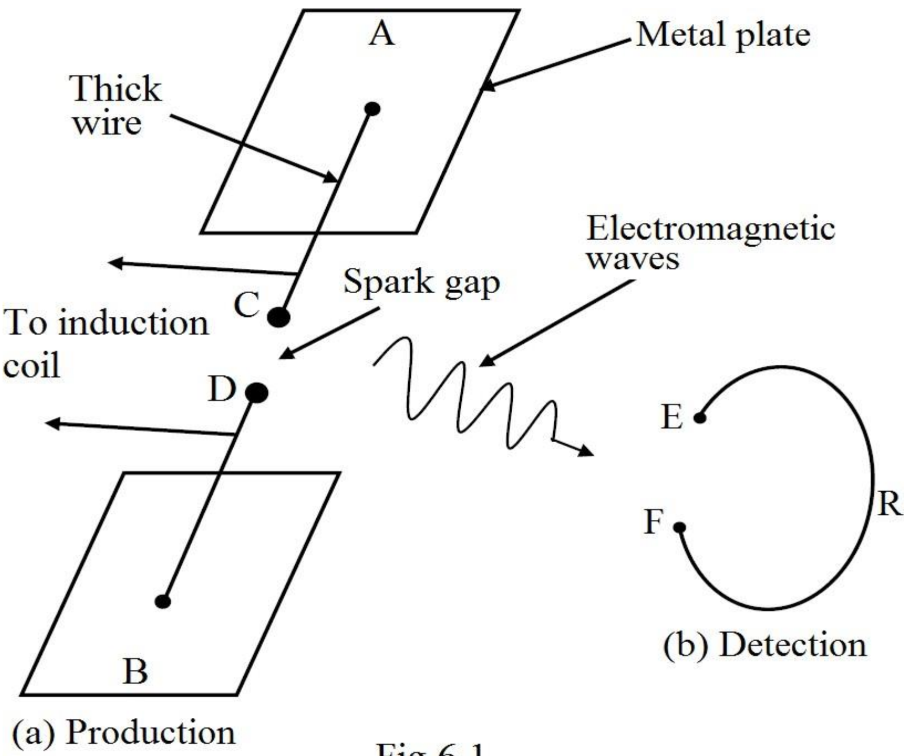


Fig 6.1

The plates A and B form a capacitor and the thick connecting wires serve as inductor. This system is equivalent to a series LC circuit.

Production:

When the D.C voltage supplied by induction coil becomes sufficiently high, the air gap between C and D gets ionized and conducting. The electrical discharge takes places between the two plates A and B. The oscillatory discharge through conducting air produces a train of electromagnetic waves.

Detection:

Hertz employed a thick wire ring connected to polished brass spheres E and F with a small gap as shown in fig (b). The brass spheres serves as the plates of a capacitor and the connecting wires as inductor. This setup acts as LC circuit with its natural frequency is given by $f = \frac{1}{2\pi\sqrt{LC}}$. When the electromagnetic waves passing through this circuit, an alternating e.m.f is induced round the wire. When the natural frequency of the detector becomes equal to the frequency of EM waves produced between C and D, resonance occurs. As a result, the amplitude of oscillations in the detector circuit increases and a spark is produced in the air gap of spheres E and F of the detector. In this way, the electromagnetic waves are detected.

❖ **Important questions**:

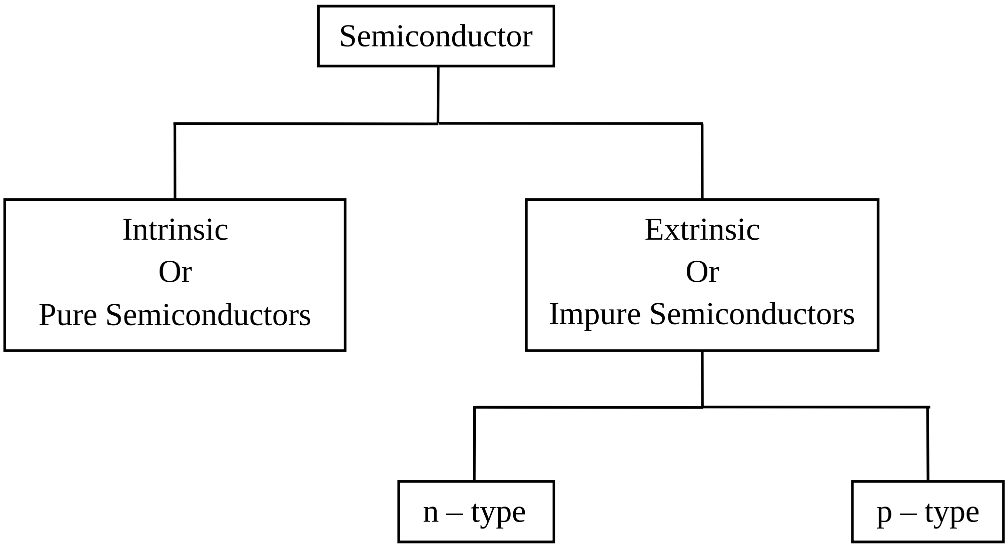
1. Explain displacement current.
2. Write Maxwell's equations in integral and differential forms.
3. Show that electromagnetic waves are transverse in nature.
4. Derive the Maxwell's electromagnetic wave equation. Show that wave velocity is equal to $v = \frac{1}{\sqrt{\mu\varepsilon}}$
5. Explain Hertz experiment to produce and detect electromagnetic waves.

Unit – IV
Chapter – 7: Basic electronics

❖ **Introduction**:

- **Semiconductor**: A substance which has resistivity in between conductors and insulators is known as semiconductor.
  Ex: Silicon, Germanium, Selenium, Carbon etc.
  - Semiconductors have negative temperature coefficient of resistance i.e., the resistance of a semiconductor decreases with the temperature and vice – versa.
  - When a suitable metallic impurity is added to a semiconductor, its current conducting properties change appreciably.

Semiconductors may be classified as



❖ **Intrinsic or Pure semiconductor**: ***A semiconductor in an extremely pure form is known as intrinsic semiconductor.***
Ex: Pure crystals like silicon, germanium are called intrinsic semiconductors.
- In an intrinsic semiconductor, the number of free electrons is always equal to the number of holes.
- The electrical conduction through a semiconductor is by both electrons and holes.
- The total current inside the semiconductor is sum of the currents due to free electrons and holes.

❖ **Extrinsic semiconductor**: At room temperature, the intrinsic semiconductor has little current conduction capability. The electrical conductivity of pure semiconductor can be increased by adding some impurity.
The semiconductor in an impure form is called an extrinsic semiconductor.
- The process of adding impurities to a semiconductor is known as ***doping***.
- If a pentavalent impurity (having 5 valance electrons) is added to the semiconductor, a large number of free electrons are produced in the semiconductor.
  Ex: Phosphorous (Z = 15), Arsenic (Z = 33), Antimony (Z = 51), Bismuth (Z = 83) etc are pentavalent impurities.

- Pentavalent impurities are called donor atoms because they donate one electron to the pure semiconductor.
- If a trivalent impurity (having 3 valance electrons) is added to the semiconductor, it creates large number of holes in the semiconductor.
  Ex: gallium (Z = 31), Indium (Z = 49)
- Trivalent impurities are called acceptor atom because it accepts one electron from semiconductor atom.
- Depending upon the type of impurity added, the extrinsic semiconductors can be divided into two classes:
  (i) n – type semiconductor
  (ii) p – type semiconductor

❖ **n – type semiconductor**: When a small amount of pentavalent impurity is added to a pure semiconductor, it is known as n – type semiconductor.

The addition of pentavalent impurity provides a large number of free electrons. The current conduction in an n – type semiconductor is mainly by free electrons. In n – type semiconductor majority charge carriers are electrons and minority charge carriers are holes.

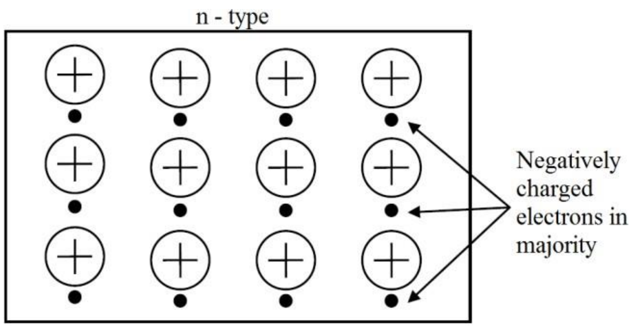


Fig 7.1

❖ **p – type semiconductor**: When a small amount of trivalent impurity is added to a pure semiconductor, it is known as p – type semiconductor.

The addition of trivalent impurity provides a large number of holes. The current conduction in p – type semiconductor is mainly by holes. In p – type semiconductor majority charge carriers are holes and minority charge carriers are electrons.

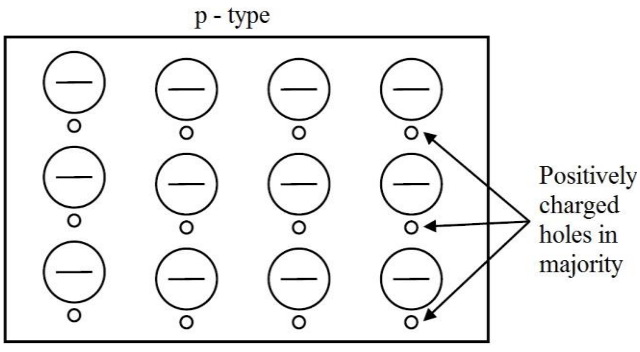


Fig 7.2

❖ **pn – junction diode**:

When a p – type material is suitably joined to n – type material, a pn – junction is formed. This is a two terminal device.

Diode Symbol: The diode symbol is shown in fig. The p – type is referred as anode and n – type is referred as cathode.

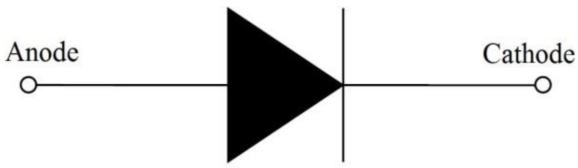


Fig 7.3

Construction:

At the instant of pn – junction formation, the free electrons near the junction in the n region diffuse across the junction into the p region and combine with holes near the junction. This creates a layer of positive charges (pentavalent ions) in n region and a layer of negative charges (trivalent ions) in p region. These two layers of positive and negative charges form the depletion region (or depletion layer). Once pn – junction is formed and depletion layer created, the diffusion of free electron stops. In other words, the depletion region acts as a barrier to the further movement of free electrons across the junction. The positive and negative charges set up an electric field.

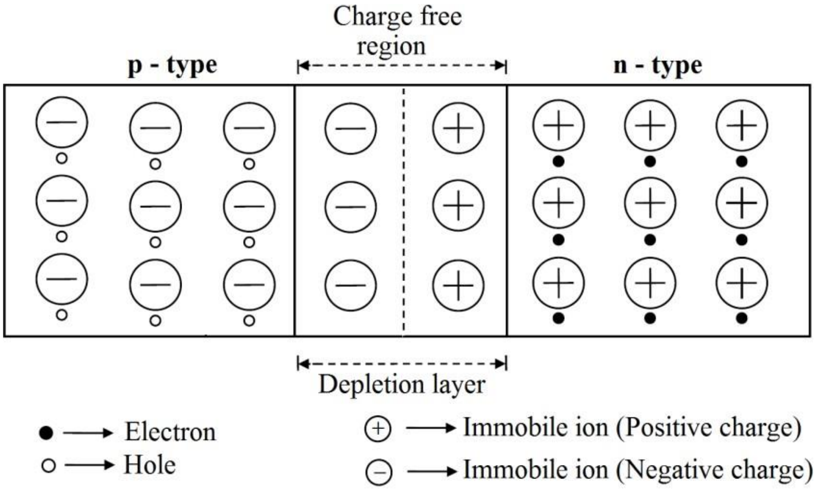
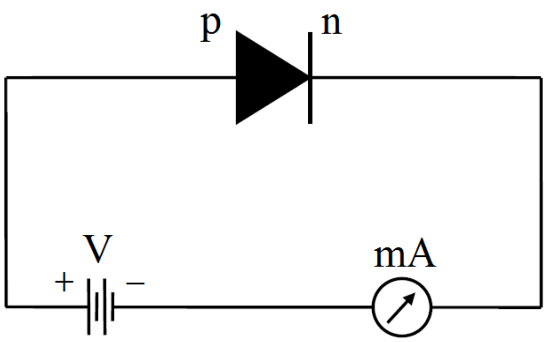


Fig 7.3

The electric field is a barrier of free electrons in the n – region. A potential difference across the depletion layer is called barrier potential V_0 . The barrier potential depends upon the several factors including the type of semiconductor material, the amount of doping and temperature.

Working:

a. **Forward bias**: ***When an external voltage is applied to pn – junction in such a direction that it cancels the potential barrier and permits the current flow is called as forward bias.*** To apply forward bias, connect positive terminal of the battery to p – type and negative terminal to n – type as shown in fig. The applied forward voltage setup an electric field which acts against the field due to potential barrier.



pn junction forward biased
Fig 7.5

(i) The potential barrier is reduced and at some forward voltage (0.1 to 0.3V), it is eliminated altogether.

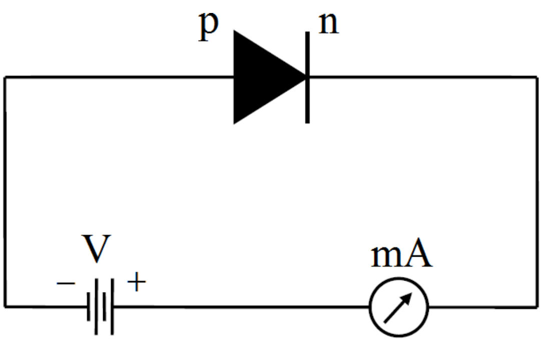
(ii) The junction offers low resistance (called forward resistance R_f) to current flow.

(iii) Current flows in the circuit due to the setup of low resistance path. The magnitude of current depends upon the applied forward voltage.

b. Reverse bias: ***When an external voltage is applied to pn – junction in such a direction that it increases the potential barrier then it is called as reverse bias.***

To apply reverse bias, connect negative terminal of the battery to p – type and positive terminal to n – type as shown in fig.

(i) The potential barrier is increased.
(ii) The junction offers very high resistance (called reverse resistance R_r) to current flow.
(iii) No current flows in the circuit due to the setup of high resistance path.



pn junction reverse biased
Fig 7.6

Thus, pn – junction is a unidirectional (one way) device which offers a low resistance when forward biased and behaves like insulator when reverse biased.

❖ **Volt – Ampere characteristics or $V - I$ characteristics**:

$V - I$ characteristics of a pn junction is a curve between voltage across the junction and the circuit current. Voltage is taken along x – axis and current along y – axis.

The characteristics are studied under the following two heads:

(i) Forward bias
(ii) Reverse bias

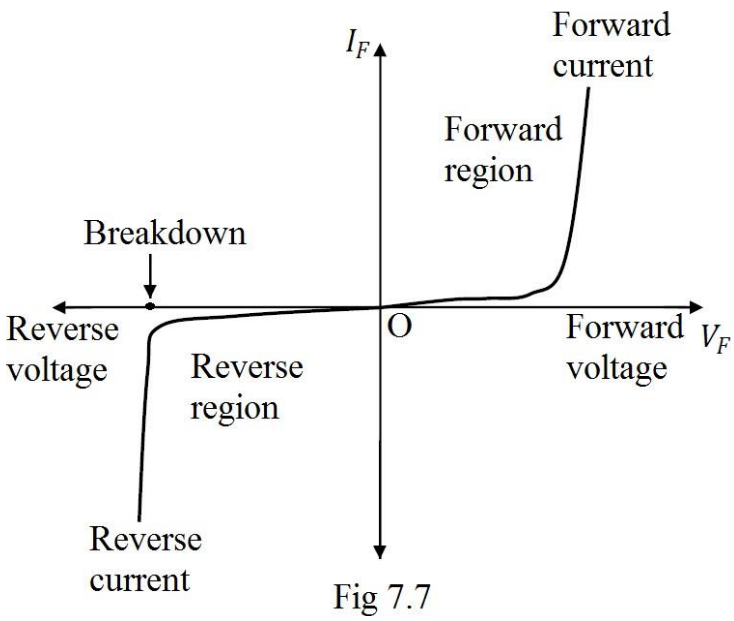


Fig 7.7

(i) **Forward bias**: In forward bias, p – type is connected to the positive terminal while the n – type is connected to the negative terminal of a battery.
The applied voltage of the diode can be varied with the help of potential divider. At some forward voltage (0.3 for Ge and 0.7 V for Si) the potential barrier altogether is eliminated and current stats flowing. This voltage is known as Threshold voltage V_{th} or cut-in voltage or knee voltage.
As the forward voltage increases beyond threshold voltage, the forward current rises exponentially. The graph is shown in the fig.

(ii) **Reverse bias**: In reverse bias, p – type is connected to the negative terminal while n – type is connected to the positive terminal of a battery.
The reverse voltage of the diode can be varied with the help of potential divider. In reverse bias, the junction resistance becomes very high and practically no current flows through the circuit. But in actual practice, a small reverse current of order μA flows in the circuit due to minority carriers. As the reverse voltage is increases from

zero, the reverse current quickly rises to its maximum or saturation value. If the reverse voltage is further increased, the kinetic energy of electrons becomes so high that they knockout electrons from the semiconductor atoms. At this stage breakdown of junction occurs and the junction is permanently destroyed. There is a sudden rise of reverse current.

❖ **<u>Zener diode</u>**:

Zener diode is a reverse biased heavily – doped silicon (or germanium) pn – junction diode which is operated in the breakdown region. Due to the higher temperature and current capability, silicon is preferred in comparison to germanium. The symbol of a zener diode is shown in fig.

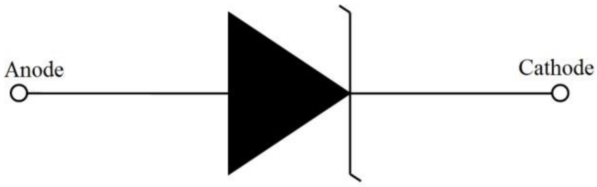
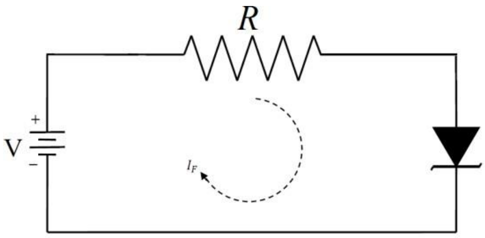


Fig 7.8

Biasing of zener diode:



(a) Forward biasing (b) Reverse biasing
Fig 7.9

1. Forward biasing: For forward biasing, the anode is connected to positive terminal of battery while the cathode is connected to negative terminal of battery. The forward is shown in fig. This biasing is generally not used.
2. Reverse biasing: For reverse biasing, the anode is connected to negative terminal while the cathode is connected to positive terminal.

$V - I$ characteristics of zener diode:

The $V - I$ characateristic curve is shown in fig. When the reverse voltage applied to pn – junction is increased from zero, the current remains very small over a long range. When the reverse voltage is made very high, reverse current increases suddenly to a large value breaking the covalent bonds near the junction.

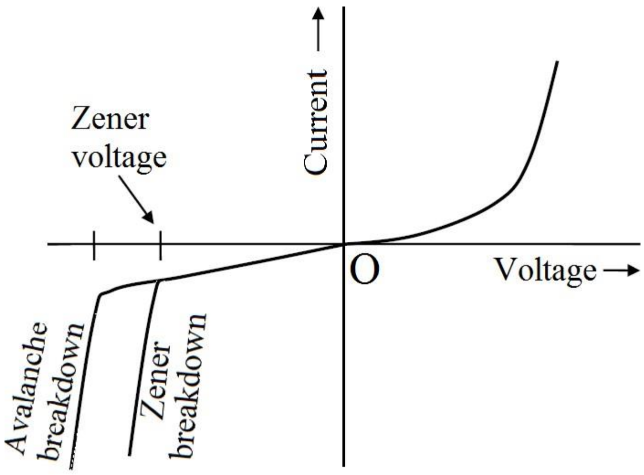


Fig 7.10

There are two mechanisms of the breakdown:

(i) **Zener breakdown**: Zener breakdown takes place in very thin junction, i.e., when both sides of junctions are very heavily doped and consequently depletion layer is narrow. When a small reverse bias voltage is applied, a reverse saturation current (zener current) is produced. This current is independent of the applied voltage and depends only on the external resistance.

(ii) **Avalanche breakdown**: This type of breakdown takes place when both sides of junction are lightly doped and consequently the depletion layer is large. This breakdown occurs at higher reverse voltages.

❖ **Transistor**:

A transistor is simply a sandwich of one type of semiconductor material between two layers of the other type. There are two types of transistors.

1. NPN transistor
2. PNP transistor

1. NPN transistor: When a layer of P type material is sandwiched between two layers of N type material, the transistor is known as NPN transistor.

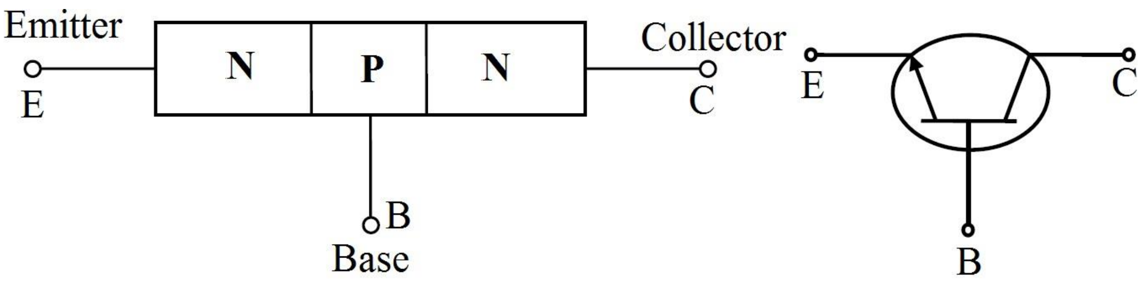


Fig 7.11

2. PNP transistor: When a layer of N type material is sandwiched between two layers of P type material, the transistor is known as PNP transistor.

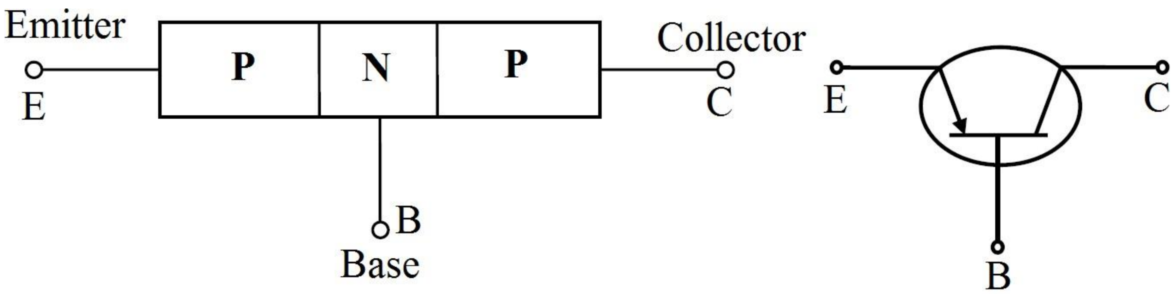


Fig 7.12

A transistor (NPN or PNP) has the following sections:

(i) Emitter: This forms the left hand section or region of the transistor. The main function of this region is to supply majority charge carriers (either electrons or holes) to the base and hence it is more heavily doped in comparison to other regions.

(ii) Base: The middle section of the transistor is known as base. This is very lightly doped and is very thin (10^{-6} m) as compared to either emitter or collector so that it may pass most of the injected charge carriers to the collector.

(iii) Collector: The right hand section of the transistor is called as collector. The main function of the collector is to collect majority charge carriers through the base. This is moderately doped.

❖ **Transistor biasing**:

In transistor biasing, the emitter – base junction is always forward biased while the collector – base junction is always reverse biased. For this purpose, a battery V_{EE} is connected between emitter and base while a battery V_{CC} is connected between collector and base.

PNP transistor:

- Emitter – base junction of PNP is forward biased by connecting the positive terminal of V_{EE} to emitter and negative terminal to base.
- Collector – base junction of PNP is reverse biased by connecting the negative terminal of V_{CC} to collector while positive terminal to base.

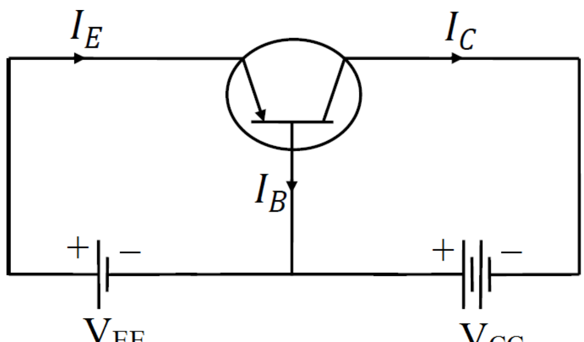


Fig 7.13

NPN transistor:

- Emitter – base junction of NPN is forward biased by connecting the negative terminal of V_{EE} to emitter and positive terminal to base.
- Collector – base junction of NPN is reverse biased by connecting the positive terminal of V_{CC} to collector while negative terminal to base.

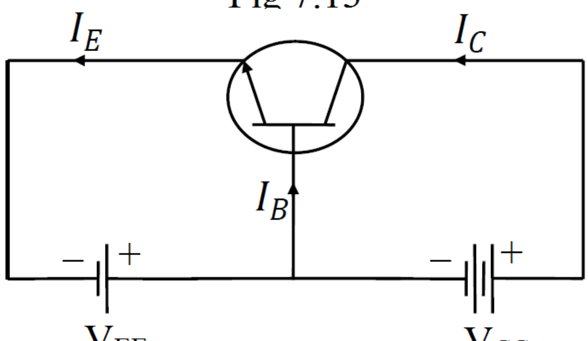


Fig 7.14

The forward biasing of emitter – base junction allows a low resistance for emitter circuit and reverse biasing of collector base junction provides high resistance in the collector circuit. In a transistor, a weak signal is introduced in low resistance circuit and the output is taken from high resistance circuit. So a transistor transfers a signal from low resistance to high resistance.

❖ **Operation of PNP transistor**:

Consider a PNP transistor with emitter – base junction as forward biased and collector – base junction reverse biased. The operation of PNP transistor is as follows:

The holes of P region (emitter) are repelled by the positive terminal of battery V_{EE} towards the base. The potential barrier at emitter junction is reduced as it is forward biased and hence the holes cross this junction and penetrate into N region. This constitutes the emitter current I_E. The width of the base region is very thin and it is lightly doped and hence only 2 to 5% of the holes recombine with the free electrons of N region. This constitutes the base current I_B which is very small. The remaining holes (95% to 98%) are able to drift across the base and enter the collector region. They are swept by the negative collector voltage V_{CC}. They constitute the collector current.

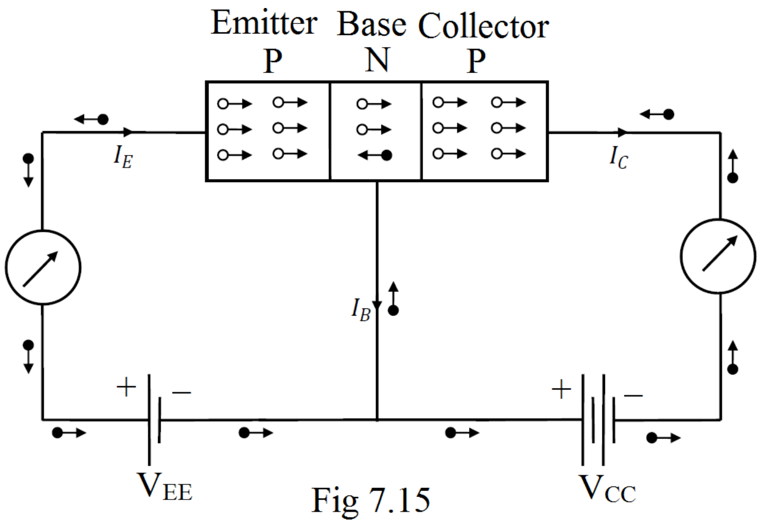


Fig 7.15

1. Current conduction within PNP transistor takes place by hole conduction from emitter to collector, i.e., majority charge carriers in a PNP transistor are holes.
2. The collector current is slightly less than the emitter current.
3. The collector current is a function of emitter current, i.e., with the increase or decrease in the emitter current, a corresponding change in collector current is observed.

Thus, only the hole current plays an important role in the operation of PNP transistor.

Therefore, $I_E = I_B + I_C$

❖ **Operation of NPN transistor**:

Consider a NPN transistor with emitter – base junction as forward biased and collector – base junction reverse biased. The operation of NPN transistor is as follows:

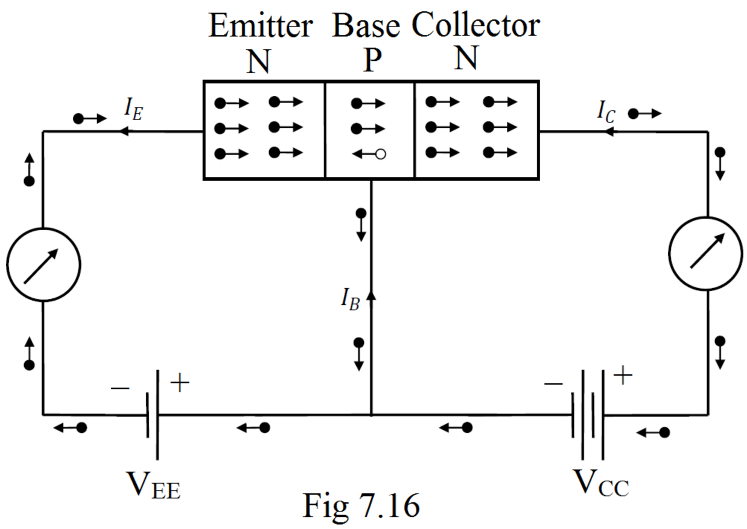


Fig 7.16

The electrons of N region (emitter) are repelled by the negative terminal of the battery V_{EE} towards the base. The potential barrier at emitter junction is reduced as it is forward biased and hence the electrons cross this junction and penetrate into P region. A few electrons combine with the holes in P region and are lost as charge carriers. Now the electrons in N region (collector region) readily swept up by the positive collector voltage V_{CC}.

For every electron flowing out the collector and entering the positive terminal of battery V_{CC}, an electron from the negative emitter battery terminal enters the emitter region. In this way electron conduction takes place continuously so long as the two junctions are properly biased.

The current conduction in NPN transistor is carried out by the electrons.

Therefore, $I_E = I_B + I_C$

❖ **<u>Transistor circuit configurations</u>**:

There are three types of transistor circuit configurations.

(1) Common – base (CB)
(2) Common – emitter (CE)
(3) Common – collector (CC)

Here, the term 'Common' is used to denote the transistor lead which is common to the input and output circuits. This is because when a transistor is connected in a circuit, four terminals are required (two for input and two for output) while a transistor has only three terminals. This difficulty is removed by making one terminal of the transistor '***common***' to both input and output terminals. The common terminal is generally grounded.

The different configurations of a PNP transistor are shown in fig.

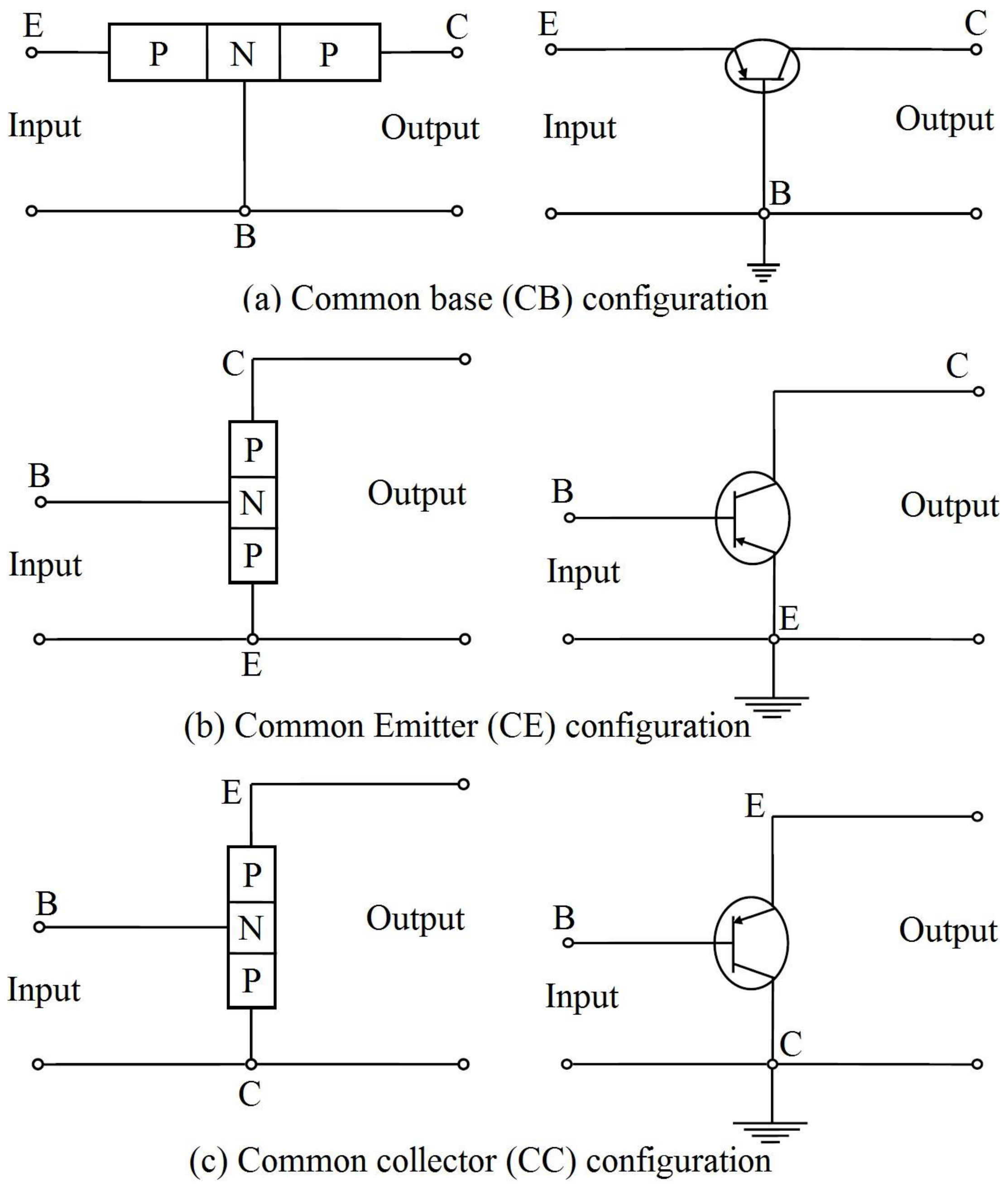


Fig 7.17

❖ **Common base configuration**:

In CB configuration, input signal is applied between emitter and base while the output is taken from collector and base. As the base is common to input and output circuits, it is called as common base configuration.

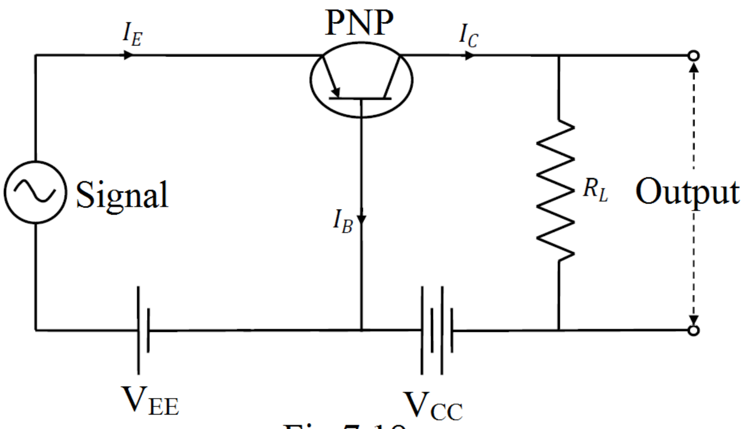


Fig 7.18

Current amplification factor α

(i) When no signal is applied

The ratio of the collector current to the emitter current is called α_{dc} of a transistor.

$\alpha_{dc} = \frac{-I_C}{I_E}$

− indicates I_C flows out of the transistor and I_E flows in.

$I_C = \alpha I_E$

$I_B = I_E - I_C$

$I_B = I_E - \alpha I_E = I_E \, 1 - \alpha$

(ii) When signal is applied

The ratio of change in collector current to the change in emitter current at constant collector base voltage is defined as current amplification factor.

$\alpha_{ac} = \frac{-\Delta I_C}{\Delta I_E}$

For all physical purposes $\alpha_{dc} = \alpha_{ac} = \alpha$ and practical values in commercial transistors range from 0.9 to 0.99

Total collector current

Total collector current consists of two parts:

(i) The current produced by majority charge carriers and its value is αI_E.

(ii) The leakage current $I_{leakage}$. This is due to the minority charge carriers across base – collector junction. This is named as I_{CBO} , i.e., collector base current with emitter open.

∴ Total collector current $I_C = \alpha I_E + I_{CBO}$

❖ **Common – emitter configuration**:

In CE configuration, the input signal is applied between base and emitter and the output is taken from collector and emitter. As the emitter is common to input and output circuits, it is called as common emitter configuration.

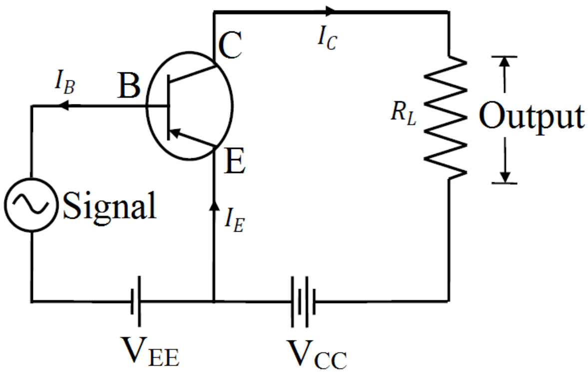


Fig 7.19

Current amplification factor β :

(i) When no signal is applied

The ratio of collector current to the base current called dc beta β_{dc} of transistor.

$$\beta_{dc} = \beta = \frac{I_C}{I_B}$$

(ii) When signal is applied

The ratio of change in collector current to the change in base current is defined as base current amplification factor.

$$\beta_{ac} = \beta = \frac{\Delta I_C}{\Delta I_B}$$

In all transistors, the base current is less than 5% of the emitter current. The value of β ranges from 20 to 500.

Total collector current

$\therefore$ Total collector current $I_C = \beta I_B + I_{CEO}$ ....(1)

I_{CEO} is the leakage current and is called collector to emitter current with base open.

We know that $I_E = I_B + I_C$ and $I_C = \alpha I_E + I_{CBO}$

$I_C = \alpha\ I_B + I_C + I_{CBO}$

$I_C\ 1 - \alpha = \alpha I_B + I_{CBO}$

$I_C = \frac{\alpha}{1-\alpha} I_B + \frac{1}{1-\alpha} I_{CBO}$ .... (2)

Comparing (1) & (2), we get

$\beta = \frac{\alpha}{1-\alpha}$ and $I_{CEO} = \frac{1}{1-\alpha} I_{CBO}$

❖ **<u>Common – collector configuration</u>**:

In CC configuration, the input signal is applied between base and collector and output is taken from the emitter. As collector is common to input and output circuits, it is called as common collector circuit.

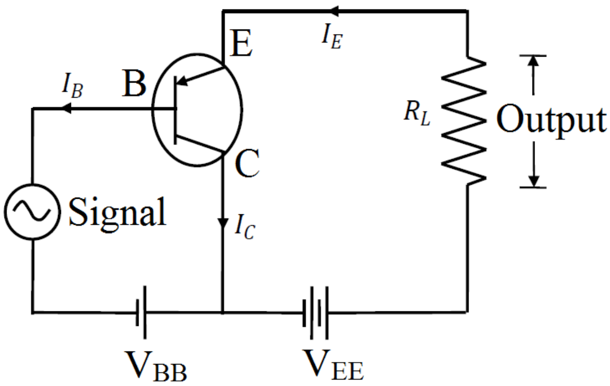


Fig 7.20

Current amplification factor γ :

(i) When no signal is applied

The ratio of emitter current to the base current is called as dc gamma γ_{dc} of the transistor.

$$\gamma_{dc} = \gamma = \frac{I_E}{I_B}$$

(ii) When signal is applied

The ratio of change in emitter current to the change in base current is known as current amplification factor γ.

$$\gamma = \frac{\Delta I_E}{\Delta I_B}$$

Total emitter current

We know that $I_E = I_B + I_C$ and $I_C = \alpha I_E + I_{CBO}$

$I_E = I_B + \alpha I_E + I_{CBO}$

$= I_B + \alpha I_E + I_{CBO}$

$I_E\ 1 - \alpha = I_B + I_{CBO}$

$$I_E = \frac{I_B}{1-\alpha} + \frac{I_{CBO}}{1-\alpha}$$

$$I_E = 1 + \beta\ I_B + 1 + \beta\ I_{CBO}$$

$$\therefore \frac{1}{1-\alpha} = 1 + \beta$$

❖ Relation between α, β and γ:

1. Relation between α and β

   $\alpha = \frac{I_C}{I_E}$ and $\beta = \frac{I_C}{I_B}$

   $I_E = I_B + I_C$ or $I_B = I_E - I_C$

   $\beta = \frac{I_C}{I_E - I_C} = \frac{I_C / I_E}{1 - I_C / I_E} = \frac{\alpha}{1-\alpha}$

   $\beta = \frac{\alpha}{1-\alpha}$ .... (1)

   $\beta\ 1 - \alpha\ = \alpha$ or $\beta - \beta\alpha = \alpha$

   $\beta = \alpha\ 1 + \beta$

   $\alpha = \frac{\beta}{1+\beta}$ .... (2)

   $1 - \alpha = \frac{1}{1+\beta}$ .... (3)

2. Relation between γ and α

   We know that $\gamma = \frac{I_E}{I_B}$ and $\alpha = \frac{I_C}{I_E}$

   $I_B = I_E - I_C$

   $\gamma = \frac{I_E}{I_E - I_C} = \frac{1}{1 - I_C/I_E} = \frac{1}{1-\alpha}$ .... (4)

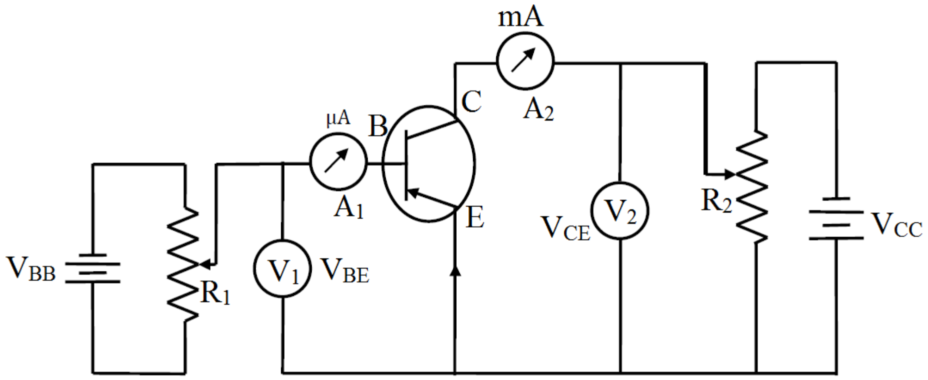
3. Relation between γ and β

   From equation (3), $1 - \alpha = \frac{1}{1+\beta}$

   Substituting this value in equation (4), $\gamma = \frac{1}{1-\alpha} = 1 + \beta$

❖ Characteristics of common emitter transistor circuit:

Consider the circuit arrangement for plotting the characteristics of a PNP transistor in CE configuration.



PNP transistor connected in common emitter configuration
Fig 7.21

We consider the following characteristics

(1) Input characteristics
(2) Output characteristics

1. **Input characteristics**:

The curve between base current I_B and base – emitter voltage V_{BE} at constant collector – emitter voltage V_{CE} represents the input characteristic.

For plotting the input characteristic, the collector – emitter voltage V_{CE} is kept fixed. The base emitter voltage V_{BE} is varied with the help of potential divider R_1 and the base current I_B is noted for each value of V_{BE}.
A graph of I_B against V_{BE} is drawn. The curve so obtained is known as input characteristic.

Input resistance: The ratio of change in base emitter voltage ΔV_{BE} to the change in base current ΔI_B at constant collector – emitter voltage V_{CE} is defined as input resistance. This is defined by r_i.

$$r_i = \frac{\Delta V_{BE}}{\Delta I_B} \quad V_{CE} = constant$$

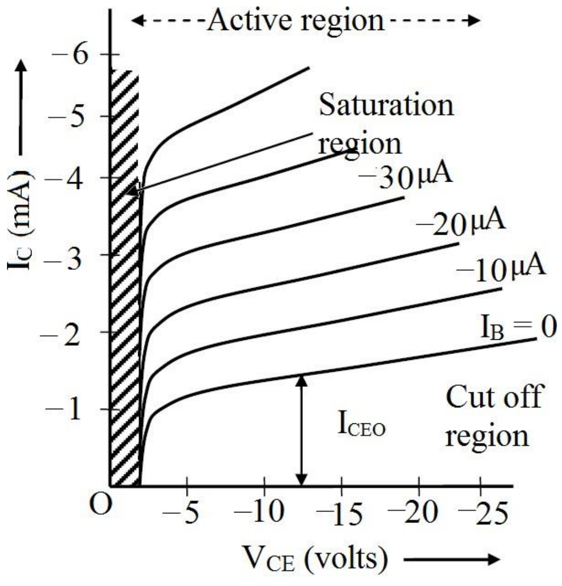


Fig 7.23

2. **Output characteristics**:

The curve between collector current I_C and collector emitter voltage V_{CE} at constant base current I_B represents the output characteristic.
For plotting output characteristic, the base current I_B is kept fixed. The collector emitter voltage V_{CE} is varied with the help of potential divider R_2 and collector current I_C is noted for each value of V_{CE}.
A graph of I_C against V_{CE} is drawn. The curve so obtained is known as output characteristic.

(i) In the active region, for small values of base current, the effect of collector voltage over collector current is small while for large base current values this effect increases.

(ii) When V_{CE} has very low value, the transistor is said to be saturated and it operates in the saturation region. In this region, the change in base current I_B does not produce a corresponding change in collector current I_C.

(iii) In the cutoff region, a small amount of collector current flows even when base current $I_B = 0$. This is called I_{CEO} . Since, main collector current is zero, the transistor is said to be cutoff. **Output resistance**: The ratio of change in collector

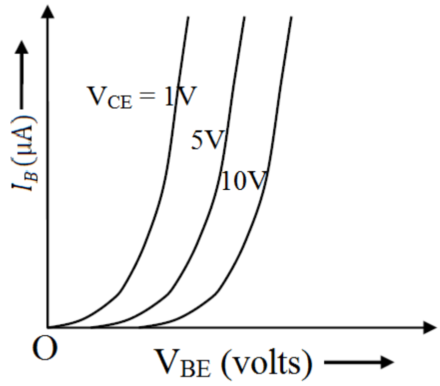


Fig 7.22

– emitter voltage ΔV_{CE} to the change in collector current ΔI_C at constant base current I_B is defined as output resistance. This is denoted by r_0.

$$r_0 = \left(\frac{\Delta V_{CE}}{\Delta I_C}\right)_{I_B = constant}$$

❖ **Hybrid parameters – Determination of hybrid parameter from transistor characteristics**:

(1) Input impedance h_{ie} : The input impedance is defined as the ratio of change in the base – emitter voltage to the change in base current at constant collector – emitter voltage V_{CE}

Mathematically, $h_{ie} = \left(\frac{\Delta V_{BE}}{\Delta I_B}\right)_{V_{CE}}$

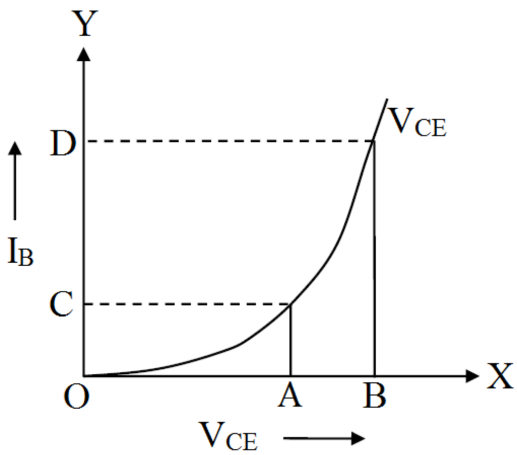


Fig 7.24

Units: Ohms (Ω)

Determination: h_{ie} can be determined from the forward characteristic curves of a transistor. Consider a forward curve between V_{BE} and I_B at constant V_{CE} . Now take slope of curve as shown in fig.

In fig $\Delta V_{BE} = AB$; $\Delta I_B = CD$

$\therefore h_{ie} = \frac{AB}{CD}\Omega$

(2) Reverse voltage ratio h_{re} : The reverse voltage ratio h_{re} is defined as the ratio of change in base – emitter voltage to the collector emitter voltage at constant base current I_B.

Mathematically, $h_{re} = \left(\frac{\Delta V_{BE}}{\Delta V_{CE}}\right)_{I_B}$

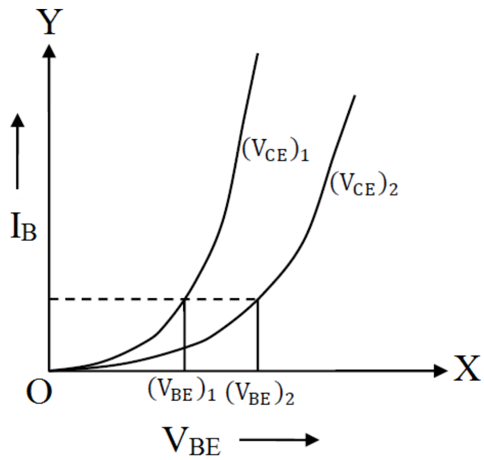


Fig 7.25

Units: No units

Determination: h_{re} can also be determined from the input characteristic curve of a transistor. Consider two forward characteristic curves at different values of I_B on the characteristic curves.

From graph,

$\Delta V_{CE} = V_{CE\,2} - V_{CE\,1}$

$\Delta V_{BE} = V_{BE\,2} - V_{BE\,1}$

$\therefore h_{re} = \frac{V_{BE\,2} - V_{BE\,1}}{V_{CE\,2} - V_{CE\,1}}$

(3) Forward current ratio h_{fe} : The forward current ratio h_{fe} is defined as the ratio of change in collector current to the change in base current at constant collector emitter voltage.

Mathematically, $h_{fe} = \left.\frac{\Delta I_C}{\Delta I_B}\right|_{V_{CE}}$

Units: No units

Determination: h_{fe} can be determined from the current characteristics curves of a transistor. Consider two output characteristic curves of different input current values of I_B. Draw a line from a particular value of V_{CE} on the characteristic curves from fig.

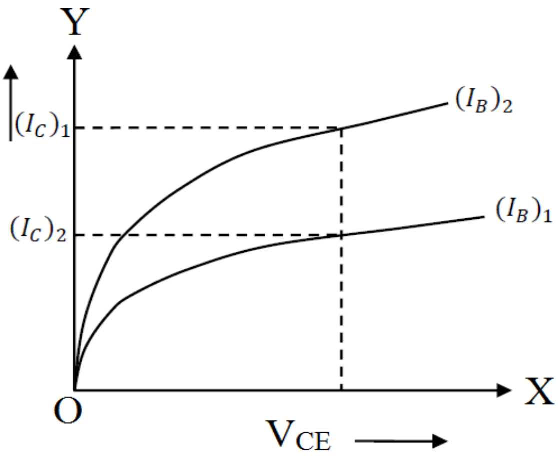


Fig 7.26

$\Delta I_C = I_{C\,2} - I_{C\,1}$

$\Delta I_B = I_{B\,2} - I_{B\,1}$

$\therefore h_{fe} = \frac{I_{C\,2} - I_{C\,1}}{I_{B\,2} - I_{B\,1}}$

(4) Output admittance h_{oe} : The output admittance h_{oe} is defined as the ratio of change in collector current to the change in collector emitter voltage at constant input current I_B.

Mathematically, $h_{oe} = \left.\frac{\Delta I_C}{\Delta V_{CE}}\right|_{I_B}$

Units: mho

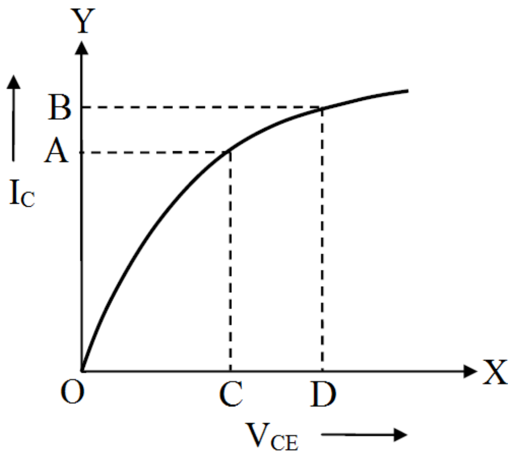


Fig 7.27

Determination: h_{oe} can be determined from the output characteristic curves of a transistor. Consider an output curve between V_{CE} and I_C at a particular value of input current I_B

In fig, $\Delta I_C = AB$

$\Delta V_{CE} = CD$

$\therefore h_{oe} = \frac{AB}{CD}\Omega^{-1}$

❖ **<u>Transistor as an amplifier</u>**:

Consider the basic circuit of a transistor amplifier. Here, the weak signal is applied between emitter – base circuit and the output is taken across the load resistor R_L connected in the collector circuit.

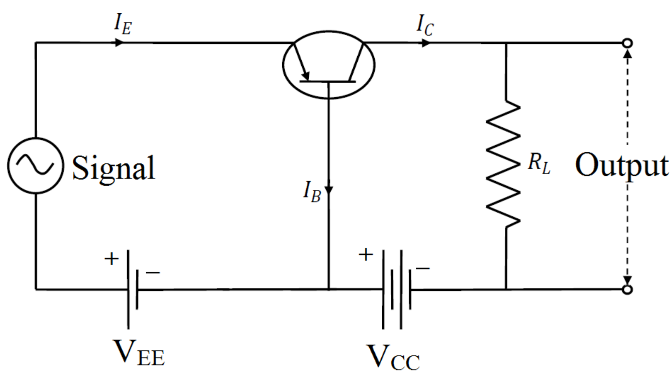


Fig 7.28

A small change in signal voltage produces an appreciable change in emitter current because the input circuit has low resistance. Now, due to the transistor action, the change in emitter current causes same change in collector current.

When the collector current flows through the load resistance R_L , a large voltage is developed across it. In this way, a weak signal is amplified.

Let a small voltage change ΔV_i between emitter and base causes a relatively large emitter – current change ΔI_E.

$$\alpha = \frac{\Delta I_C}{\Delta I_E}$$

$$\Delta I_C = \alpha \Delta I_E$$

The change in output voltage across the load resistor $\Delta V_0 = R_L \times \Delta I_C$

$$= R_L \times \alpha \Delta I_E$$

The voltage amplification $A = \frac{\Delta V_0}{\Delta V_i}$

Will be greater than unity and the transistor acts as an amplifier. If the dynamic resistance of the emitter junction is r_e , then $\Delta V_i = r_e \times \Delta I_E$

$$A = \frac{R_L \times \alpha \Delta I_E}{r_e \times \Delta I_E}$$

$$A = \frac{\alpha R_L}{r_e}$$

Voltage gain A_V :

This is defined as the ratio of change in output voltage ΔV_{CE} to the change in input voltage ΔV_{BE} when transistor is connected in common emitter configuration.

$$A_V = \frac{\Delta V_{CE}}{\Delta V_{BE}}$$

$$= \frac{\text{Change in output current} \times \text{effective load}}{\text{Change in input current} \times \text{input resistance}}$$

$$= \frac{\Delta I_C \times R_L}{\Delta I_B \times R_i} = \beta \frac{R_L}{R_i}$$

Power gain A_P :

This is defined as the ratio of output signal power to input power signal.

$$A_p = \frac{\Delta I_C{}^2 \times R_L}{\Delta I_B{}^2 \times R_i} = \frac{\Delta I_C}{\Delta I_B} \cdot \frac{\Delta I_C \times R_L}{\Delta I_B \times R_i}$$

$$= \beta \times A_V$$

Power gain = current gain × voltage gain

❖ **<u>Important Questions</u>**:

1. What is a pn – diode? Explain its working.
2. Draw the $V - I$ (or $I - V$) characteristics of a P – N junction diode and explain them
3. What is a zener diode? Explain the operation of a zener diode.
4. Draw and explain $V - I$ characteristics of zener diode.
5. Explain the operation (or working) of PNP transistor.
6. Explain the operation (or working) of NPN transistor.
7. Explain the CE characteristics of a transistor.
8. Describe CB, CE and CC configurations of a transistor.
9. Describe transistor hybrid parameters
10. How does transistor work as amplifier?

Unit – V
Digital electronics

❖ **Introduction**:

The digital electronics is a branch of electronics which deals with the generation, processing and storage of digital signals. The digital electronics was first invented by George Boole. The algebra which is used in digital electronics is called as Boolean algebra. In digital electronics, any information can be represented in terms of 0's and 1's. It is represented by a bit. The digital operations are two state operations. These two states are expressed as

HIGH – LOW
ON – OFF
TRUE – FALSE
YES – NO
1 – 0

❖ **Some basic terms**:

1. **Number system**: A number system is a code. For each distinct quantity there is an assigned symbol. Hence a number system relates quantities and symbols.
2. **Base or Radix**: It is the number of digits or basic symbols in a number system. The decimal system has a base ten because it uses ten digits.
3. **Bit**: It is an abbreviated form of binary digit.
   1100 – four binary digits or 4 bits.
   Note: 1 nibble = 4 bits
   1 byte = 8 bits

❖ **Number systems**:

In digital electronics, we use different types of number systems. They are

1. Decimal number system (0 to 9)
2. Binary number system (0 or 1)
3. Octal number system(0 to 7)
4. Hexadecimal number system (0 to 9, A, B, C, D, E, F)

1. Decimal number system:

- The decimal system has a radix or base of 10.
- It contains 10 digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.
- The position value (or weights) in the system are powers of ten.

Examples:

$8765 = 8 \times 10^3 + 7 \times 10^2 + 6 \times 10^1 + 5 \times 10^0$

5 – Least significant digit (LSD)

8 – Most significant digit (MSD)

$12.91 = 1 \times 10^1 + 2 \times 10^0 + 9 \times 10^{-1} + 1 \times 10^{-2}$

.... $10^3\ 10^2\ 10^1\ 10^0\ .\ 10^{-1}\ 10^{-2}\ 10^{-3}$ ....

↑ Decimal point

2. Binary number system:

- The binary number system has a base or radix of 2.
- It contains two digits: 0 and 1
- The position value (or weight) in the system are powers of 2.

Examples:

$(1001)_2 = 1 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = (9)_{10}$

Extreme right digit – Least significant bit (LSB)

Extreme left digit – Most significant bit (MSB)

$(1101.11)_2 = 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 + 1 \times 2^{-1} + 1 \times 2^{-2}$

$= 8 + 4 + 0 + 1 + 0.5 + 0.25 = (13.25)_{10}$

3. Octal number system:

- The octal system has a base or radix of 8.
- It contains 8 digits: 0, 1, 2, 3, 4, 5, 6, 7.
- The position value (or weights) in the system are powers of 8.

Ex: $(258)_8 = 2 \times 8^2 + 5 \times 8^1 + 8 \times 8^0$

$= 2 \times 64 + 40 + 8 = (176)_{10}$

4. Hexadecimal number system:

- Hexadecimal system has a base or radix of 16.
- It contains 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F
- The position value (or weights) in the system is powers of 16.

Examples:

$(1F)_{16} = 1 \times 16^1 + 15 \times 16^0$

$= 16 + 15 = (31)_{10}$

❖ **Decimal to binary conversion:**

Decimal number to binary number:

Step 1: Divide progressively the decimal number by 2 and write down the remainder after each division.

Step 2: Continue this process till you get quotient of 0 and remainder 0 or 1.

Step 3: The remainders taken in the reverse order form the binary number.

This method is called double – dabble method because it requires successive divisions by 2.

Example: convert the decimal number 37 to its equivalent binary number.

2	37	
2	18	- 1
2	9	- 0
2	4	- 1
2	2	- 0
2	1	- 0
	0	- 1

(LSB) ↑ (MSB) $(100101)_2$

$(37)_{10} = (100101)_2$

Decimal fraction to binary number:

When a decimal number is a fraction, its binary equivalent is obtained by multiplying the number continuously by 2, recording each time a carry in its integer position.

Example: convert 0.435 into binary number

0.435 ×2 = 0.870 with a carry of 0 (MSB)

0.870 × 2 = 1.74 = 0.74 with a carry of 1

0.74 × 2 = 1.48 = 0.48 with a carry of 1

0.48 × 2 = 0.96 with a carry of 0 (LSB)

$(0.435)_{10} = (0.11)_2$

- **Binary to decimal conversion**:

The conversion of a binary number into decimal equivalent is somewhat an easy process.

The decimal equivalent is obtained by multiplying the individual digits by the ascending powers of 2 (2^0, 2^1, 2^2, ...) moving from right to left and then adding them.

Example:

$1101 = 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$

$= 8 + 4 + 0 + 1$

$= 13$

$(1101)_2 = (13)_{10}$

To convert binary fraction into its decimal equivalent we multiply each digit in the fraction successively 2^{-1}, 2^{-2}, 2^{-3}, ...

Example:

$0.1011 = 1 \times 2^{-1} + 0 \times 2^{-2} + 1 \times 2^{-3} + 1 \times 2^{-4}$

$= \frac{1}{2} + 0 + \frac{1}{8} + \frac{1}{16}$

$= \frac{8+0+2+1}{16} = \frac{11}{16}$

$= 0.6875$

$(0.1011)_2 = (0.6875)_{10}$

- **Laws of Boolean algebra**:

1. **Laws of Complementation (NOT Laws)**:

   The term complement means to invert. The symbol is an over bar.

   Law 1: $0 = 1$ Law 2: $1 = 0$

   Law 3: If $A = 0$, then $A = 1$ Law 4: If $A = 1$, then $A = 0$

   So, for a logic variable X, we have $X \cdot X = 0 \Rightarrow X + X = 1$

2. **OR Laws:**

   The OR operation is represented by + sign. If A and B are inputs and Y is output, then OR operation is written as

   $$Y = A + B$$

   So, 0 + 0, 0 + 1, 1 + 0 and 1 + 1

   From these expressions, it is clear that if both the inputs are 0, then output will be zero and if any input or both inputs is 1, then the output will be 1.

   The four OR laws are

   Law 1: $A + 0 = A$ Law 2: $A + 1 = 1$

   Law 3: $A + A = A$ Law 4: $A + A = 1$

3. **AND Laws**:

   The AND operation is represented by multiplication. $Y = A \cdot B$ shows the AND laws.

   So, $0 \cdot 0 = 0$, $0 \cdot 1 = 0$,

   $1 \cdot 0 = 0$, $1 \cdot 1 = 1$

So, it is clear that if any input is zero or both inputs are zero, then the output will be zero while if both the inputs are 1, the output will be one.

The four AND laws are

Law 1: $A \cdot 0 = 0$ Law 2: $A \cdot 1 = A$

Law 3: $A \cdot A = A$ Law 4: $A \cdot A = 0$

4. **Commutative Laws**:

   There are two commutative laws. These laws allow change in the position of variables in OR and AND expressions.

   These are

   Law 1: $A + B = B + A$

   Law 2: $A \cdot B = B \cdot A$

5. **Associative Laws**:

   There are two associative laws. These laws allow removal of bracket from logical expression and regrouping of variables.

   These are

   Law 1: $A + B + C = A + B + C$

   Law 2: $A \cdot B \cdot C = A \cdot B \cdot C$

6. **Distributive Laws**:

   There are three distributive laws. These laws show that we can expand expressions by multiplying terms as in ordinary algebra. The distributive laws are

   Law 1: $A \cdot B + C = A \cdot B + A \cdot C$ Law 2:
   $A + B \cdot C = A + B \cdot A + C$ Law 3: $A + A$
   $\cdot B = A + B$

7. **Absorptive Laws**:

   There are three absorptive laws. These are

   Law 1: $A + A \cdot B = A$

   Law 2: $A \cdot A + B = A$

   Law 3: $A \cdot A + B = A \cdot B$

❖ <u>**De Morgan's laws**</u>:

Theorem 1: ***The complement of the sum of two or more variables is equal to the product of the complement of the variables, i.e., for a two input gate we can write***

$$\boldsymbol{A + B = A \cdot B}$$

De Morgan's first theorem can be represented by fig.

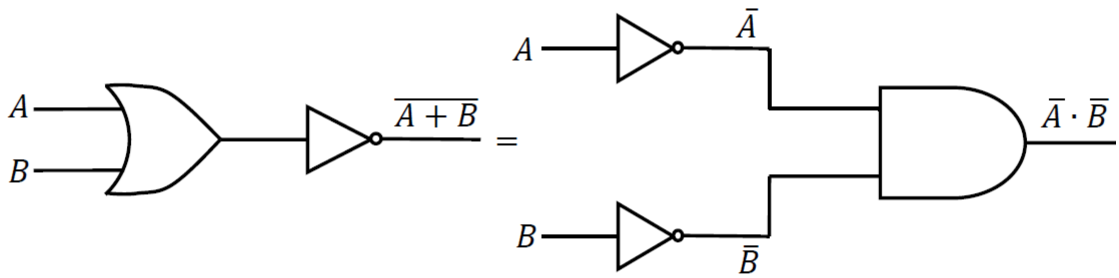


Fig 8.1

Proof:

We have four cases

a. When $A = 0$ and $B = 0$

L.H.S = $A + B = 0 + 0 = 0 = 1$

R.H.S = $A \cdot B = 0 \cdot 0 = 1 \cdot 1 = 1$ Hence

$A + B = A \cdot B$

b. When $A = 0$ and $B = 1$

L.H.S = $A + B = 0 + 1 = 1 = 0$

R.H.S = $A \cdot B = 0 \cdot 1 = 1 \cdot 0 = 0$ Hence

$A + B = A \cdot B$

c. When $A = 1$ and $B = 0$

L.H.S = $A + B = 1 + 0 = 1 = 0$

R.H.S = $A \cdot B = 1 \cdot 0 = 0 \cdot 1 = 0$ Hence

$A + B = A \cdot B$

d. When $A = 1$ and $B = 1$

L.H.S = $A + B = 1 + 1 = 1 = 0$

R.H.S = $A \cdot B = 1 \cdot 1 = 0 \cdot 0 = 0$ Hence

$A + B = A \cdot B$

Theorem 2: ***The complement of the product of two or more variables is equal to the sum of the complements of the variables, i.e.,***

$$\boldsymbol{A \cdot B = A + B}$$

De Morgan's second theorem can be represented by fig.

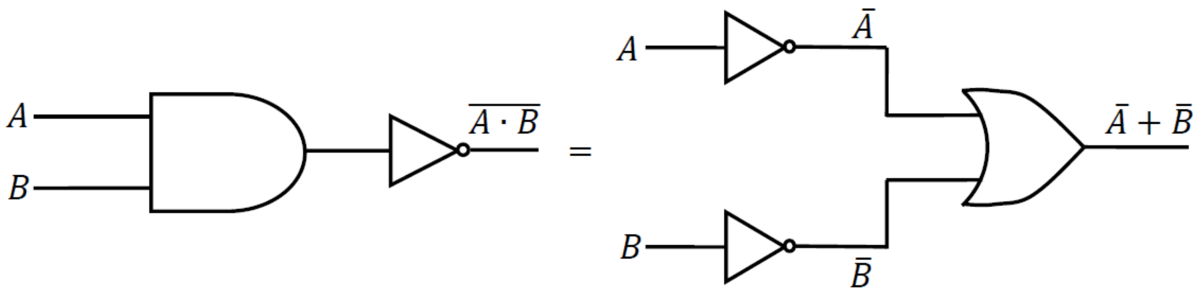


Fig 8.2

Proof:

We have four cases

a. When $A = 0$ and $B = 0$

L.H.S = $A \cdot B = 0 \cdot 0 = 0 = 1$

R.H.S = $A + B = 0 + 0 = 1 + 1 = 1$

Hence $A \cdot B = A + B$

b. When $A = 0$ and $B = 1$

L.H.S = $A \cdot B = 0 \cdot 1 = 0 = 1$

R.H.S = $A + B = 0 + 1 = 1 + 0 = 1$

Hence $A \cdot B = A + B$

c. When $A = 1$ and $B = 0$

L.H.S = $A \cdot B = 1 0 = 0 = 1$

R.H.S = $A + B = 1 + 0 = 0 + 1 = 0$

Hence $A \cdot B = A + B$

d. When $A = 1$ and $B = 1$

L.H.S = $A \cdot B = 1 1 = 1 = 0$

R.H.S = $A + B = 1 + 1 = 0 + 0 = 0$

Hence $A \cdot B = A + B$

Hence, in every case left hand side of the expression is equal to the right hand side of the expression. Therefore, the theorem is proved.

❖ **Logic gates**:

Circuits which are used to process digital signals are called logic gates. Gate is a digital circuit with one or more inputs but only one output.

Logic gates are of two types – Combinational and sequential.

In combinational gates, the output at any instant depends upon the inputs at that instant. Here the previous input does not have any effect on the output.

Examples: AND, OR, NOT, NAND, NOR and XOR are examples of combinational gates

In sequential gates, the output depends upon the order or sequence in which the inputs are applied.

Examples: Flip – flops, counters and Registers are examples of sequential gates.

❖ **Basic logic gates**:

The most basic logic gates are AND, OR and NOT gates.

- **OR gate**:

An OR gate has two or more input signals but only one output signal. It is called OR gate because the output is high if any or all of the inputs are high. The symbolic representation of a two input OR gate is shown in fig (a). Fig (b) shows its electrical equivalent which explains the logic behind any OR gate.

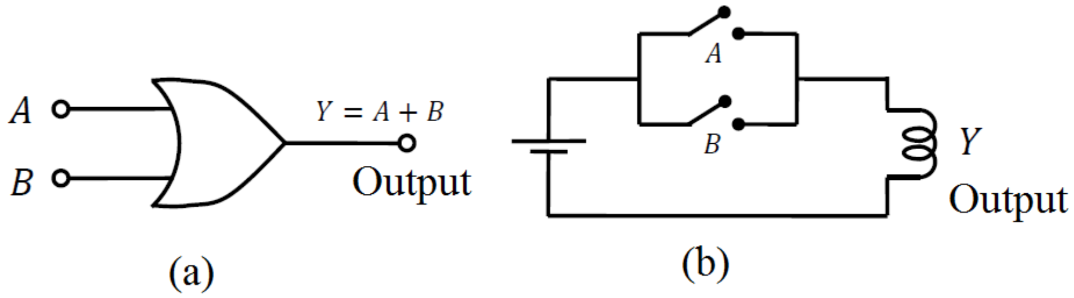


Fig 8.3

OR gate is represented by equation $Y = A + B$ and reads as Y equals A OR B and not as A plus B.

Truth table:

Input		**Output**
A	***B***	***Y = A + B***
0	0	0
0	1	1

1	0	1
1	1	1

The truth table of OR gate shows all the input output possibilities of a logic gate.
(i) When both inputs (A and B) are zero (switches are open), the output Y is zero.
(ii) When A is in logic state 0 (switch A is open) but B is in logic state 1 (switch B is closed), output Y is in logic state 1 (lamp is ON).
(iii) When A is in logic state 1 (switch A is closed) but B is in logic state 0 (switch B is open), output Y is in logic state 1 (lamp is ON).
(iv) when both the switches are in logic state 1 (switches are closed) then the output Y will be in logic state 1 (lamp is ON).

- **<u>AND gate</u>**:
  An AND gate has two or more input signals but only one output signal. It is called AND gate because it provides output HIGH when all the inputs are HIGH. The symbolic representation of a two input AND gate is shown in fig.

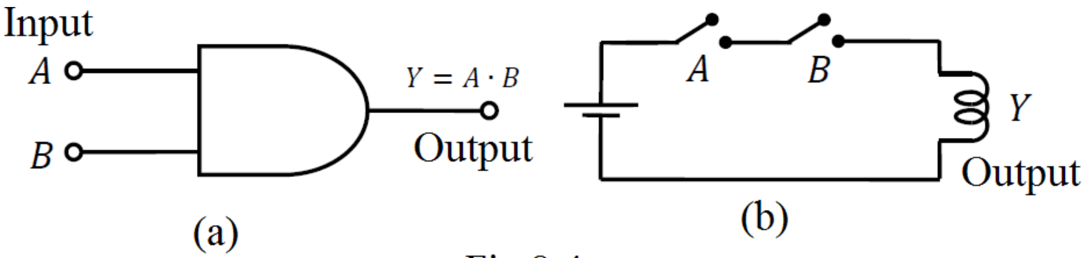


Fig 8.4

AND gate is represented by the equation $Y = A \cdot B$ and reads as Y equals A AND B and not as A multiplied by B.

Truth table:

Input		**Output**
$\boldsymbol{A}$	$\boldsymbol{B}$	$\boldsymbol{Y = A \cdot B}$
0	0	0
0	1	0
1	0	0
1	1	1

The truth table of AND gate shows all the input output possibilities of a logic gate.
(i) When both inputs (A and B) are zero (switches are open), the output Y is zero.
(ii) When A is in logic state 0 (switch A is open) but B is in logic state 1 (switch B is closed), output Y is in logic state 0 (lamp is OFF).
(iii) When A is in logic state 1 (switch A is closed) but B is in logic state 0 (switch B is open), output Y is in logic state 0 (lamp is OFF).
(iv) when both the switches are in logic state 1 (switches are closed) then the output Y will be in logic state 1 (lamp is ON).

- **<u>NOT gate</u>**:
  The NOT gate is a gate with only one input and one output. The NOT gate is also called an inverter because the output state is always opposite to that of input state. That is when the input is HIGH, the output is LOW and vice-versa.

NOT gate is represented by the equation $Y = A$ when A is input
The symbolic representation of NOT gate is shown in fig.

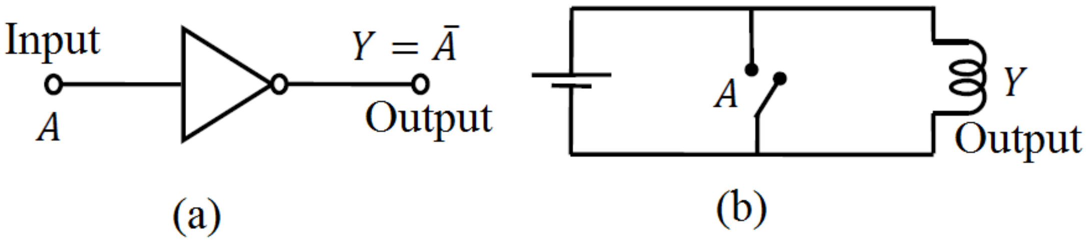


Fig 8.5

Truth table:

Input	**Output**
A	$Y = A$
0	1
1	0

Input	**Output**
A	$Y = A$
LOW	HIGH
HIGH	LOW

❖ **NAND gate**:

NAND gate is a combination of AND gate and a NOT gate. The symbol of a NAND gate is shown in fig. Here the bubble on the output reminds us of the inversion after AND operation.

NAND gate is represented by the equation $Y = AB$

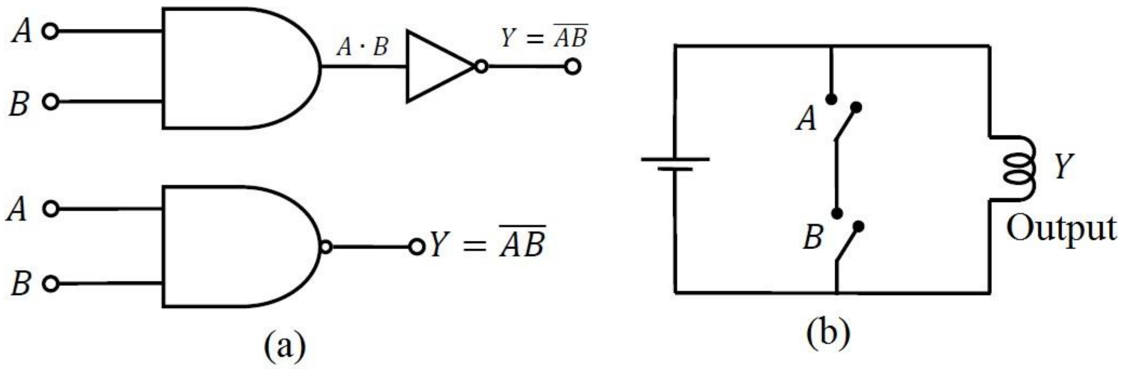


Fig 8.6

Truth table:

Input		Output
A	B	$Y = AB$
0	0	1
0	1	1
1	0	1
1	1	0

(i) When $A = 0$, $B = 0$ then $AB = 0$ and $AB = 1$ (ii) When $A = 0$, $B = 1$ then $AB = 0$ and $AB = 1$ (iii) When $A = 1$, $B = 0$ then $AB = 0$ and $AB = 1$ (iv) When $A = 1$, $B = 1$ then $AB = 1$ and $AB = 0$

- **<u>NAND gate as a universal gate</u>**:

The NAND gate is called a universal gate since any logic gate (OR, AND, NOT) can be built by using NAND gate only.

As NOT gate

If two inputs of NAND gate are connected together, then we get a NOT gate as shown in fig.

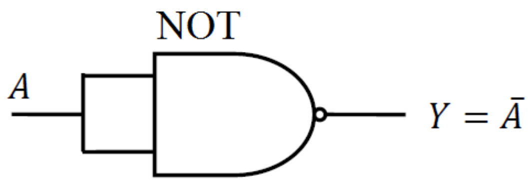


Fig 8.7 (a)

As AND gate

The AND gate can be produced by connecting two NAND gates in series as shown in fig.

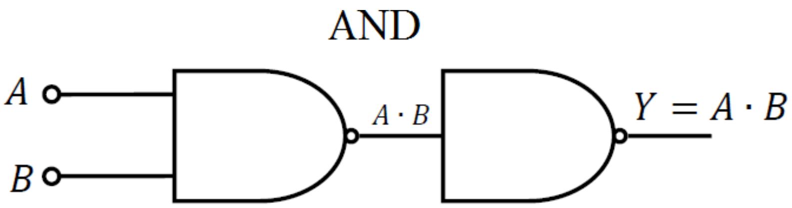


Fig 8.7 (b)

As OR gate

OR gate can be produced by three NAND gates as shown in fig. It is important to mention here that OR function may not be clear from this figure because De Morgan's theorem is needed to prove that $A \cdot B = A + B$

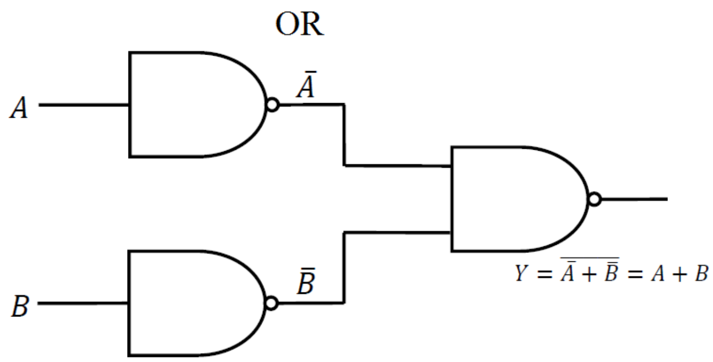


Fig 8.7 (c)

- **<u>NOR gate</u>**:

NOR gate is a combination of OR gate and a NOT gate. The symbol of NOR gate is shown in fig. Here the bubble in the output signifies that inversion takes place after OR operation.

NOR gate is represented by the equation $Y = A + B$

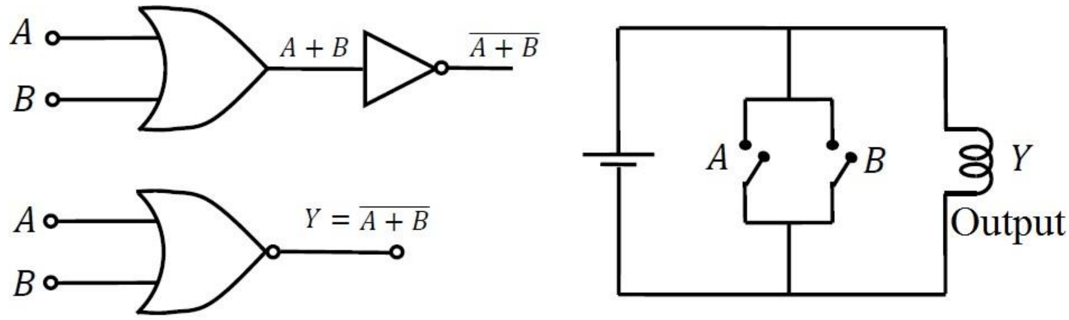


Fig 8.8

Truth table:

Input		Output
A	B	$Y = A + B$
0	0	1
0	1	0
1	0	0
1	1	0

(i) When $A = 0$, $B = 0$ then $A + B = 0$ and $A + B = 1$ (ii) When $A = 0$, $B = 1$ then $A + B = 1$ and $A + B = 0$ (iii) When $A = 1$, $B = 0$ then $A + B = 1$ and $A + B = 0$ (iv) When $A = 1$, $B = 1$ then $A + B = 1$ and $A + B = 0$

❖ **NOR gate as universal gate**:

NOR gate is a universal gate because it can be used to perform the basic logic functions AND, OR, and NOT.

As NOT gate

When the inputs of NOR gate are tied together, the output is $A + A$ as shown in fig. By De Morgan's theorem this is equivalent to A (*i.e.*, $A + A = A$). This is the function of NOT gate.

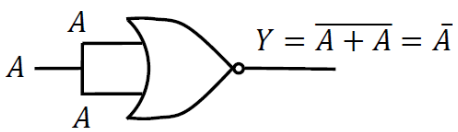


Fig 8.9 (a)

As AND gate

AND gate can be made out of three NOR gates as shown in fig. Here two NOR gates are used to invert the inputs and third gate is used to combine the inverted inputs.

The output $A \cdot B$ is a function of AND gate.

This can be proved with the help of De Morgan's law. $A + B = A \cdot B = A \cdot B$

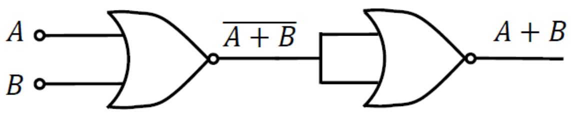


Fig 8.9 (b)

As OR gate

This gate can be produced by connecting output of a NOR gate to a NOT gate.

The output of NOR gate is $A + B$. This is inverted by NOT gate to give $Y = A + B$.

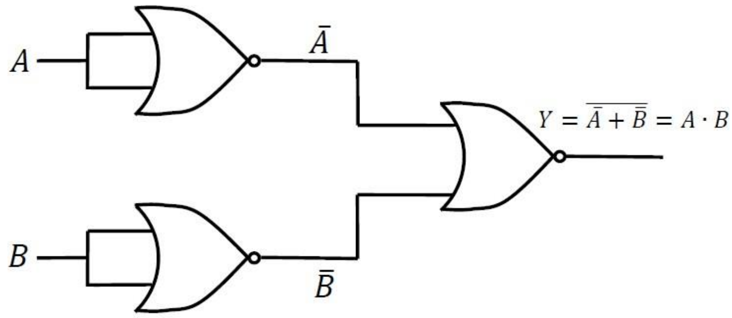


Fig 8.9 (c)

❖ **<u>Exclusive OR gate (XOR Gate)</u>**:

The EXCLUSIVE – OR gate, commonly written as EX-OR gate, is a two – input, one-output gate. The symbol of a XOR gate is shown in fig (a) & (b). The exclusive OR operation is denoted by ⊕. Hence the output is given by

$$Y = A \oplus B = AB + A\ B$$

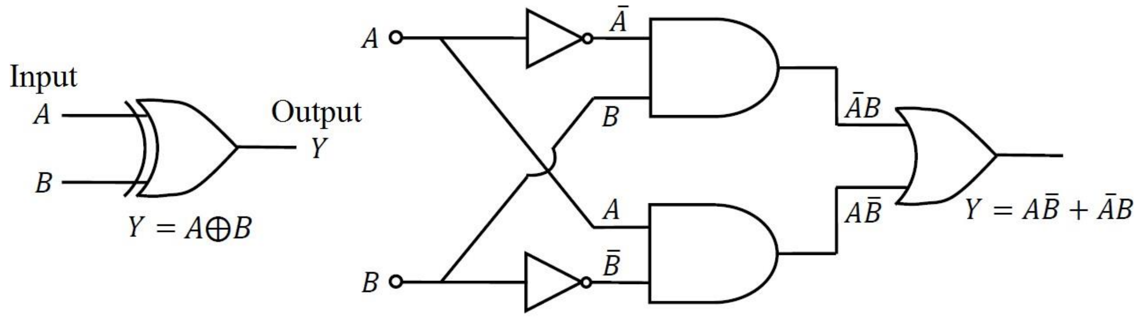


Fig 8.10

The circuit of XOR gate is shown in fig. The bulb will glow when $A = 1$ or $B = 1$ but not both.

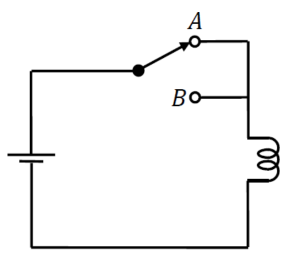


Fig 8.11

Truth table:

Input		Output
A	B	$Y = A \oplus B$
0	0	1
0	1	0
1	0	0
1	1	0

(i) When $A = 0$, $B = 0$, $Y = 0 \cdot 1 + 1 \cdot 0 = 0 + 0 = 0$

(ii) When $A = 0$, $B = 1$, $Y = 0 \cdot 0 + 1 \cdot 1 = 0 + 1 = 1$

(iii) When $A = 1$, $B = 0$, $Y = 1 \cdot 1 + 0 \cdot 0 = 1 + 0 = 1$

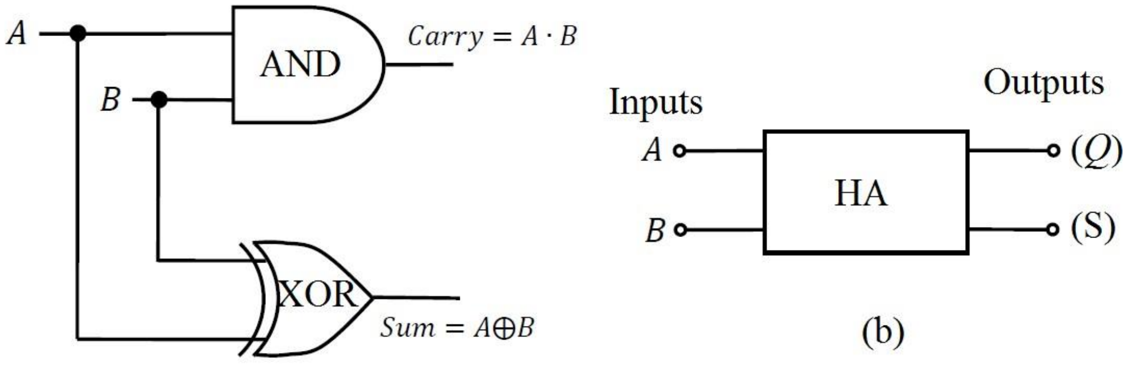
(iv) When $A = 1$, $B = 1$, $Y = 1 \cdot 0 + 0 \cdot 1 = 0 + 0 = 0$

It is clear from the truth table that output is 1 only when the inputs are different and output is 0 when the inputs are same.

❖ Half adder:

A *half-adder* is an arithmetic circuit block that can be used to add two bits. It has two inputs that represent the two bits to be added and two outputs, with one producing the SUM output and the other producing the CARRY.

The fig. shows the circuit of a half adder. It consists of an exclusive – OR gate and an AND gate. The output of the exclusive – OR gate is called the SUM, while the output of the AND gate is called the CARRY.



(a) (b)

Fig 8.12

Truth table:

The truth table of a Half – adder is developed by writing the truth – table output of AND gate in **Carry** column and output of truth table of ex – OR gate in **Sum** column.

Input		Output	
A	***B***	**CARRY *Q***	**SUM *S***
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

(i) When $A = 0$ and $B = 0$

Carry $Q = A \cdot B = 0 \cdot 0 = 0$

Sum $S = A \oplus B = 0 \cdot 0 = 0$

(ii) When $A = 0$and $B = 1$

Carry $Q = A \cdot B = 0 \cdot 1$

Sum $S = A \oplus B = 0 \otimes 1 = 1$

(iii) When $A = 1$ and $B = 0$

Carry $Q = A \cdot B = 1 \cdot 0 = 0$

Sum $S = A \oplus B = 1 \oplus 0 = 1$

(iv) When $A = 1$ and $B = 1$

Carry $Q = A \cdot B = 1 \cdot 1 = 1$

Sum $S = A \oplus B = 1 \oplus 1$

❖ Full adder:

A *full adder* circuit is an arithmetic circuit block that can be used to add three bits to produce a SUM and a CARRY output. The full adder circuit overcomes the limitation of the half-adder, which can be used to add two bits only. The circuit and symbol of full adder is shown in fig.

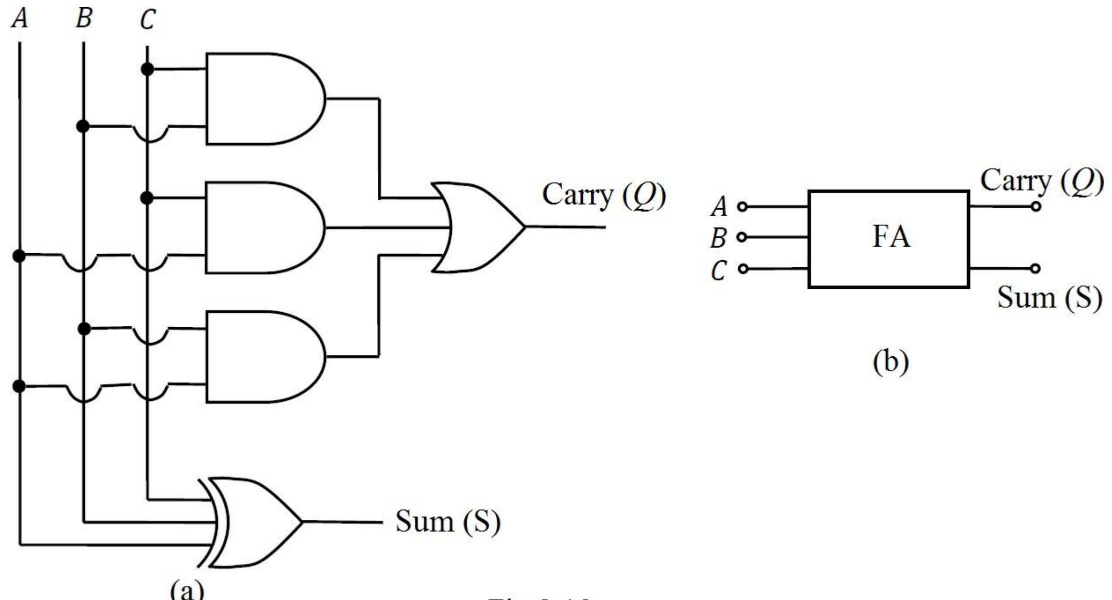


Fig 8.13

Truth table:

Input			**Output**	
A	**B**	**C** Q'	**Carry** Q	**Sum** S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

❖ **Important Questions**:

1. Explain different number systems.
2. State the laws of Boolean algebra.
3. State and prove De Morgan's laws.
4. Describe the basic logic gates with truth tables.
5. Explain a NAND gate. Show that NAND gate is a universal gate.
6. Explain a NOR gate. Show that NOR gate is a universal gate
7. Describe XOR gate with truth table.
8. Explain the construction and working of Half adder.
9. Explain the construction and working of Full adder.